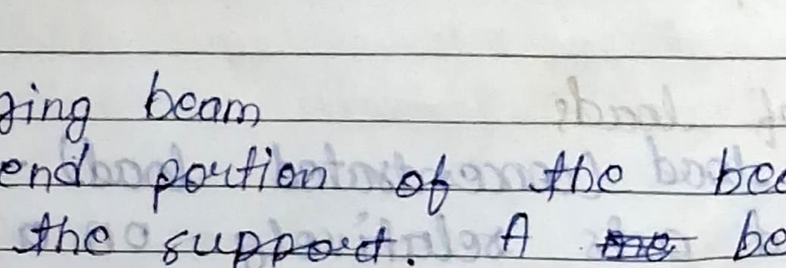
MOM

Beam – A structural member subjected to external forces or couples at right ∠s to the longitudinal axis is called ~~a~~ beam. OR
Beam is structural member is vertically loaded.

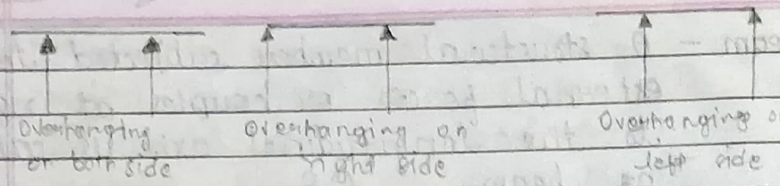
· Types of Beams

1.) Simply supported beam
– A beam which is freely supported on the walls or columns at its both ~~on~~ the ends.

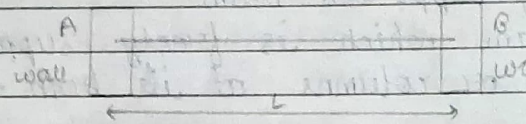
2.) Cantilever beam
– A beam fixed at one end and free at the other.



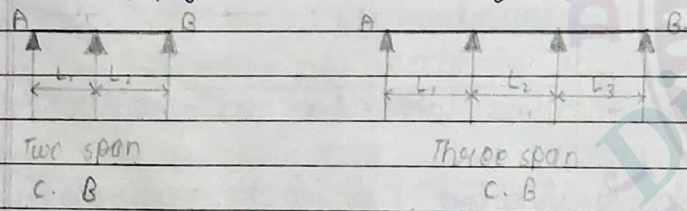
3.) Overhanging beam
If the end portion of the beam extends beyond the support. A ~~the~~ beam may be overhanging on one side or on both side



4.) Fixed Beam (constrained beam, built-in beam, encastre beam)
A beam whose both the ends are rigidly fixed in walls



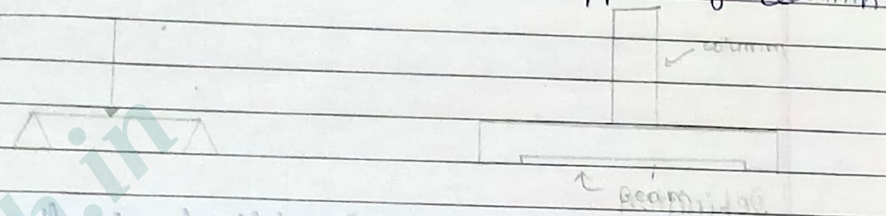
5.) Continuous Beam
A beam which is supported on more than two supports (i.e., at least three supports). The end supports of a continuous beam may be simply supported or fixed.



Types of loads

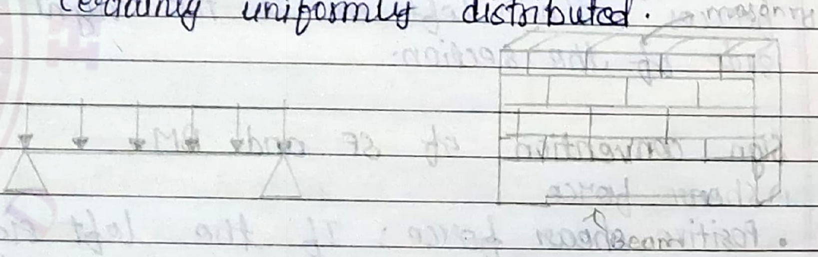
1) Point load (concentrated load) - This type of load acts relatively on a smaller area. The force is concentrated at one location, creating a high intensity at that point.

Ex- force exerted by a chair or a table leg on the supporting floor or load exerted by a beam on a supporting collumn



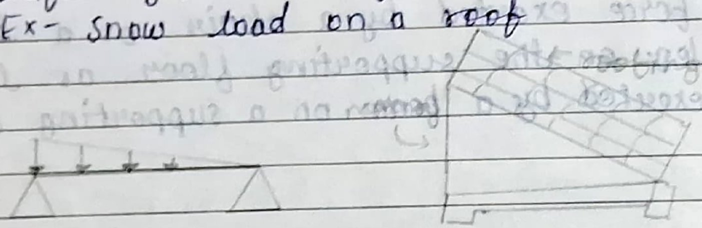
2) UDL (Uniformely distributed load)
It is spread over a large area. Its magnitude is designated by its intensity (N/m or kN/m).

Ex - If a floor slab is supported by beams, the load of the slab on the beams is certainly uniformly distributed.



3) UVL (Uniformly Varying load):
a load that is distributed along the length of a beam or structure but varies linearly throughout the length of the beam. The load's intensity increases or decreases

uniformly / unit length.
Ex- Snow load on a roof



Definition

- A shear force (SF) is defined as the algebraic sum of all vertical forces, either to the left ~~side~~ or to the right hand side of the section.

- A bending moment (BM) is defined as the algebraic sum of moments of all the forces either to the left or to the right hand side of the section.

Sign convention of SF and BM

1.) Shear force

- Positive shear force: If the left side of the section experiences an upward force and the right side experiences a downward force, the shear force is considered positive.

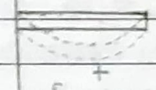
- Negative shear force: If the left side experiences a downward force and the right side experiences an upward force, the shear force is considered negative.

* Positive shear: Upward on left ↑
downward on right ↓

Negative shear: Downward on left ↓
upward on right ↑

2.) Bending Moment (M):

- Positive Bending moment: If the beam bends such that the concave side is at the bottom (like a "smile" shape), the moment is considered positive.

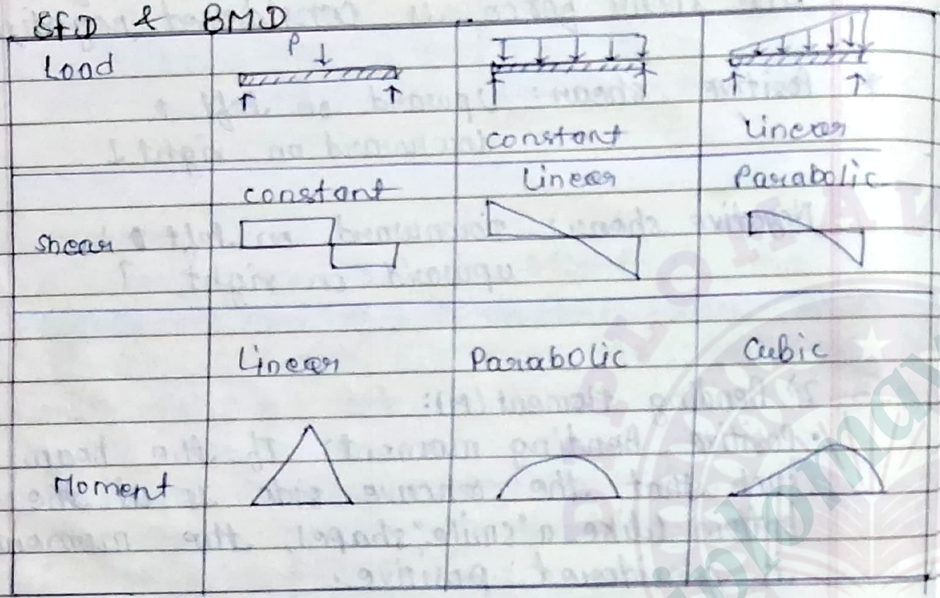


- Negative Bending moment: If the beam bends such that the concave side is at the top (like a "frown" shape), the moment is considered negative.

Concave

* • Positive moment: Sagging (+)
• Negative moment: Hogging (−)

SFD & BMD



Load	P	constant	Linear
Shear	constant	Linear	Parabolic
Moment	Linear	Parabolic	Cubic

CH- 2

Load — The external force or forces applied to a material or structure. The forces can cause deformation, stress, and strain in the material, which can lead to failure if the material is unable to withstand the load.

Types of loads

1.) Tensile load:- A force that stretches or pulls a material, causing elongation (e.g., a rope being pulled).

2.) Compression load:- A force that compresses or shortens a material, causing it to shrink (e.g., a column under weight).

3.) Shear load:- A force that acts ∥ to a material's surface, causing one part to slide over another (e.g., scissors cutting paper).

4.) Impact load:- A sudden force applied over a short time, causing rapid deformation or stress (e.g., a hammer hitting a nail).

(Ch-2)

Stress – The internal resistive force per unit area, offered by a body against deformation is known as stress.

The external force acting on the body is called load or force. The load is applied on the body while the stress is induced in the material of the body.

MLT^{-2} Stress (σ) $= \frac{P}{A}$ or $\frac{F}{A}$ where, P = external force, A = cross-sectional area

$= \frac{\text{External applied force}}{\text{cross-sectional area}}$

S.I unit of stress (σ) $= N/m^2 = Nm^{-2}$

The force is expressed in Newton (N) and area is expressed in m^2. Hence, unit of stress becomes as Nm^{-2}, if unit of load or force and unit of area is in MKS unit then unit of stress become kgf/m^2

- Types of stress:-

~~The stresses may be~~

i) Normal stress (longitudinal stress)

ii) Shear stress (Tangential stress).

iii) volumetric stress (Bulk stress).

iv) Torisonal stress

i) Normal stress :- Normal stress is the stress which acts in a direction ⊥ to the area.

OR

The restoring force / unit area ⊥ to the surface of the body.

- It is represented by σ (sigma)

Normal stress — (I) Tensile stress
(II) Compressive stress

(I) Tensile stress :- The stress induced in a body, when subjected to two equal and opposite pulls as a result of which there is an increase in the length. $\sigma = \frac{P}{A}$

(II) Compressive stress :- The stress induced in a body, when subjected to two equal and opposite pushes as a result of which there is a decrease in the length of the body. $T_c = \frac{P}{A}$

ii) Shear stress – The stress induced in a body when subjected to two equal and opposite forces which are acting tangentially across the resisting section, as a result of which the body tends to shear off across the section.

OR

when the shape of a body changes due to deforming force. $T_s = \frac{P}{A}$

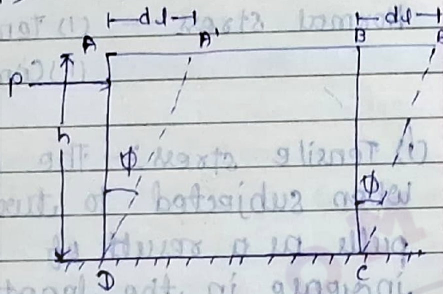
- It is denoted by (τ) tao

if a force 'P' is applied on the block which is fixed at the bottom, the face ABCD will be distorted to A'B'CD through an angle ϕ

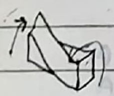
Then shear stress (ϕ) is given by:-

$\phi = \frac{AA'}{AC}$

$\phi = \frac{dl}{h}$



iii) Volumetric stress — When the vol. of body changes due to the deforming force



iv) Torsional stress — stress produced when a material or structure is twisted, such as in shafts subjected to torque.

type of mech. stress that occurs when equal forces act on all sides of a body, causing a change in its vol. but not its shape.

Strain:- When a body is subjected to some external force, there is some change ~~of~~ in dim. of the body.

The ratio of the change of dim. (ie length, shape or vol.) of the body to the original dim. (i.e length, shape or vol.) of the body.

→ Strain is unit less and dimensionless.

→ strain = change in dim. / original dim.

→ represented by ε or ϵ

The strain may be:-

1.) Tensile strain
2.) Compressive strain } Longitudinal strain.
3.) shear strain
4.) volumetric strain

i.) Tensile strain:- $\frac{\text{Increase/change in length}}{\text{Original length}} = \frac{dl}{l}$

ii) Compressive strain:- $\frac{\text{Decrease/change in length}}{\text{original length}} = \frac{dl}{l}$

iii) Shear strain:- It is a measure of how much an object deforms when subjected to a shear force. ~~dendo~~ denoted by γ $\tan\theta = \frac{x}{L}$ or $\theta = \frac{x}{L}$

lateral displacement / fixed layer of object

iv) Volumetric strain:- It is the ratio of change in volume of a material relative to its original volume due to an applied stress.

$$\varepsilon_v = \frac{\Delta V}{V_0}$$

Elasticity:- The ability of a material to return to its original shape and size after being ~~subjected~~ removal of deforming force.

$\sigma = E \cdot \varepsilon$

Plasticity:- The ability of a material to undergo permanent deformation without breaking and fracturing when subjected to an external force or stress.
→ It occurs when the applied stress exceeds the material's elastic limit or yield strength.
→ The deformation is irreversible and does not return to the original shape after the force is removed.

Elastic limit:- The maximum stress from which an elastic body will recover its original state after the removal of the deforming force.
OR
There is a limiting value of force upto and within which the deformation completely dissappear on the removal of the force.

Elastic fatigue:- The loss of strength of the material due to repeated strains on the material.

Hook's Law:- Hook's law states that when a material is loaded within elastic limit, the stress is $\propto$ to strain produced by the stress.
This means the ratio of the stress to the corresponding strain is a constant, within the elastic limit.

This constant is known as modulus of elasticity or modulus of rigidity.

i.e:- Stress $\propto$ strain
stress = constant × strain
$\frac{\text{Stress}}{\text{strain}}$ = costant Modulus of elasticity

Modulus of elasticity:- The ratio of stress to the corresponding strain produced within elastic limit.
i.e. Modulus of elasticity (E) = $\frac{\text{stress}}{\text{strain}} = \frac{\sigma}{e}$

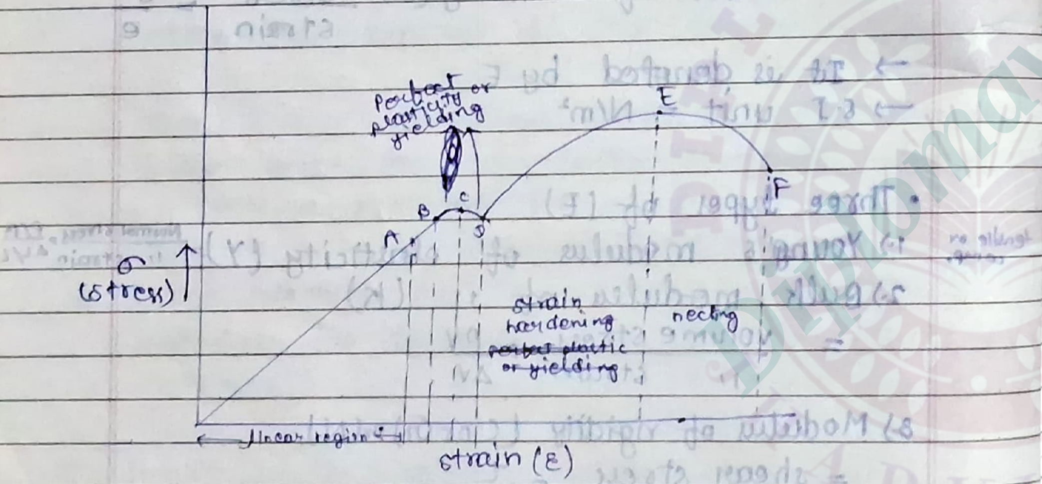
→ It is denoted by E.
→ S.I unit = N/m^2

• Three types of (E)
1.) Young's modulus of elasticity (Y) = $\frac{\text{Normal stress}}{\text{" strain}} = \frac{F/A}{\Delta L/L}$ (tensile or comp.) (stress)
2.) Bulk modules of " (K)
= $\frac{\text{volume stress}}{\text{" strain}} = \frac{PV}{\Delta V}$
3.) Modulus of rigidity (C or G) or (N)
= $\frac{\text{shear stress}}{\text{" strain}} = \frac{\tau}{\phi}$

Stress-strain diagram
Stress-strain diagram is a behaviour of material when it is subjected to load.

In this diagram, stresses are plotted along the vertical axis and as a result of these stresses, corresponding strains are plotted along the horizontal axis.

In the diagram given below, you can see the diffⁿ mark points on the curve. It is because, when a ductile material like mild steel is subjected to a tensile test, then it passes various stages before fracture.



For ductile material

Important points

A → limit of proportionality - upto this point hook's law point

B → Elastic limit → It is the point upto which strain produced in body is fully recoverable.

C → upper yield point - maximum stress before permanent deformation.

D → lower yield point - Began to deform plastically.

E → Ultimate point - maximum stress which occur in the material.

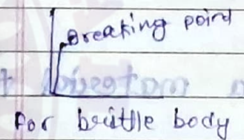
F → Fracture (Rupture) point - point at which the mild steel fracture.

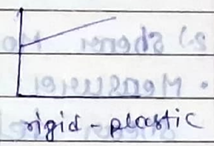
CD = plastic limit

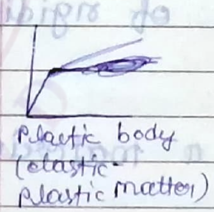
DE = strain hardening

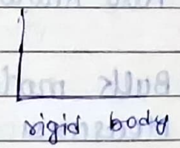
EF = Necking

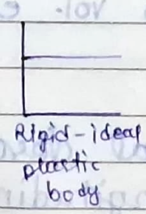
[Note:- rubber is brittle Because it does not give plastic deformation

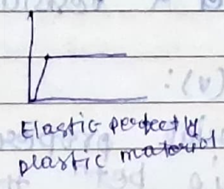


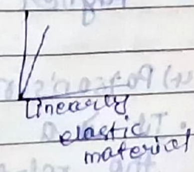












Elastic constants – They are parameters that describe the relationship betn stress and strain in materials during elastic deformation.

1.) Young's Modulus (E):
2.) shear " (G)
3) Bulk " (K)
4) Poisson's Ratio (V)

1) Young's Modulus (E):
- Measures the stiffness of a material in response to tensile stress.
- Higher value indicate greater stiffness

2) shear Modulus (G):
- Measures the resistance of a material to shear deformation
- Also known as the modulus of rigidity

3) Bulk modulus (K):
- Measures the resistance of a material to uniform compression
- Higher value ∝ resistance to vol. change.

4) Poisson's ratio (V):
- It is a material property that describes the relationship betn the longitudinal strain and the lateral strain when a material is subjected to tensile or compressive stress within elastic limit.

> Represents the tendency of a material to expand or contract laterally when stretched or compressed axially.

> When a material is stretched in one direction (longitudinal strain), it tends to contract in ⊥ direction (lateral strain)

$$\nu = -\frac{\text{lateral strain}}{\text{longitudinal strain}}$$

Note: For most materials, poisson's ratio ranges from 0 to 0.5

1) v = 0, – means no lateral strains
– deforms in one direction
2) v = 0.5, – means material is incompressible
– lateral strain = longitudinal strain
in metal betn 0.25 to 0.34

- – Poisson's ratio – occur in some special materials called auxetic materials, which expand laterally when stretched.

- also known as fatigue limit. The max^m stress a mate can withstand for infinite no. cyclic load without

Relation betn K, G and E

- Relation betn E and G

$E = 2G(1+\mu)$

$\Rightarrow \frac{E}{2G} - 1 = \mu$ ——— (1)

- Relation betn E and K

$E = 3K(1-2\mu)$

Putting eq (1)

$E = 3K\left(1 - 2\left(\frac{E}{2G} - 1\right)\right)$

$\Rightarrow E = 3K\left(1 - \frac{2E}{2G} + 2\right)$

$\Rightarrow \frac{E}{3K} = 3 - \frac{E}{G}$

$\Rightarrow \frac{E}{3K} = \frac{3G - E}{G}$

$\Rightarrow EG = 9KG - 3KE$

$\Rightarrow EG + 3KE = 9KG$

$\Rightarrow E(3K+G) = 9KG$

$\Rightarrow \boxed{E = \frac{9KG}{3K+G}}$

Linear strain (longitudinal strain)

– measure of deformation along the direction of the applied force.

linear strain $(\varepsilon) = \Delta L / L_0$

ΔL = change in length, L_0 = original length

Lateral strain – measure of deformation ⊥ to the direction of applied force

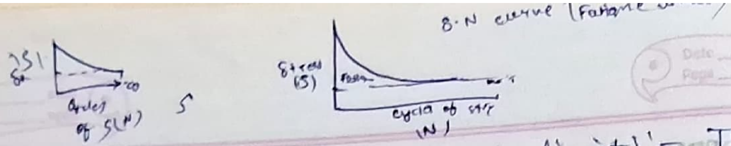
lateral strain $(\varepsilon') = \Delta W / W_0$

ΔW = change in width, W_0 = original width

$\nu = -\frac{\varepsilon'}{\varepsilon}$

Fatigue → Refers to the phenomenon where materials fail under repeated when subjected to cyclic loading or repeated stress over time. Even if the applied force is lower than the material's ultimate tensile strength

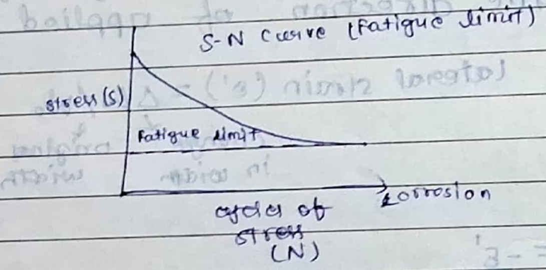
Ex – Aircraft, Bridges, pipelines, automotive components



Endurance limit (Fatigue limit):- The maximum stress level a material can withstand for an infinite no. of cyclic load without ~~experiencing~~ experiencing fatigue failure theoretically.

> E.L is not a universal property and varies depending on the material.

> E.L → steel and ferrous alloys (Yes)
→ Al and Non- "ferrous" (No)

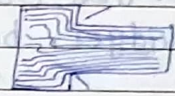


S-N curve (Wohler curve) represents the relationship betn the stress amplitude and the no. of cycles to failure.

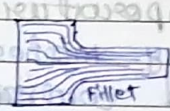
Stress concentration - It is a phenomenon where the stress within a material or structure becomes significantly higher in specific localized areas ~~due to~~ compare to surrounding areas due to the presence of geometric irregularities or material discontinuities.

Causes:
- Geometric features:- sharp corners, holes, Notches, grooves, fillets
- Material Defects:- cracks, inclusions, voids,

Effect:-
> Increased risk of failure
> fatigue



Solutions:-
> Design modifications
> Material selection
> Surface Treatments
> Stress relief techniques



FOS (Factor of safety) or safety factor
It is a design principle used to ensure that a structure or material can withstand with unexpected loads, material defects, or variations in operating conditions.

$$\text{FoS} = \frac{\text{Ultimate strength or Failure strength}}{\text{Working stress or Applied load}}$$

- Purpose — Prevent failure — Ensure reliability — Account for imperfections — Compensate for concentrations — Enhance safety.

· Typical Values:
- value of FoS depends on the appli., material and potential consequences of failure.
· for critical comp. (e.g. - aerospace, medical devices), FoS is high (e.g. 5 or more)
· For less critical or routine structures (e.g. - bridges, buildings), (1.5 to 3).

Temperature stresses

Every material expands when tempⁿ rises and contracts when tempⁿ falls. Tempⁿ stresses refers to the internal forces or stress that develop within a material due to changes in tempⁿ.
When tempⁿ changes of a material, its atoms or molecules move more energetically, leading to expansion or contraction of the material.

$$\Delta l \propto (t_2 - t_1) \times l$$

$$\Delta l = \alpha (t_2 - t_1) \times l$$

↑ coefficient of thermal expansion

$$\Delta l = \alpha \Delta t\, l$$

Coefficient of thermal expansion

Different mate. expand or contract at diffⁿ rates when the tempⁿ changes.

$$\alpha = \frac{\varepsilon}{\Delta T} \qquad \Delta l \propto \Delta T\, l \qquad \Delta l = \alpha \Delta T\, l \qquad \varepsilon = \alpha \Delta T$$

The rate at which a mate. expands or contracts is described by its CTE.

high CTE = larger dim. changes

$$\frac{\Delta l}{l} = \alpha \Delta t$$

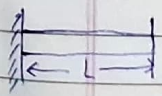
$$\text{strain} = \Delta t \times \alpha$$

$$\varepsilon = \Delta t \times \alpha \qquad \alpha = \frac{\varepsilon}{\Delta t}$$

T.s in uniform bars:-

1) Fully restrained bar:-

Let us consider a bar of uniform cross-sectional area 'A' and length 'L' are constrained at both end. By increasing the tempⁿ of the bar by ΔT, if the bar where free to expand it would have increased to length of L+ΔL. However, this increase in length ΔL is completely restraining. Hence the restraining t.s will be induced in the bar.



∴ ~~The~~ Thermal strain $(\varepsilon) = \Delta L / L$

$$\Rightarrow \varepsilon = \frac{\alpha \Delta t\, \cancel{l}}{\cancel{l}}$$

$$\Rightarrow \varepsilon = \alpha \Delta t$$

Thermal stress $(\sigma) = E \times \alpha \times \Delta T$

$\Rightarrow \sigma = E \times \varepsilon$ $\left[\because E = \frac{\sigma}{\varepsilon},\ \sigma = E \times \varepsilon\right]$

Thermal stress $(\sigma) = E \times \alpha \times \Delta T$

(ii) Partially restrained bar:-

Consider a bar which is fixed at one end and can move by distance of $\Delta L'$ ($\Delta L' < \Delta L$). In this case, while the bar is expanding the bar is free to expand by an amount $\Delta L'$ but is restrained in expanding further by rest of amount $(\Delta L - \Delta L')$.

Thus,

Thermal strain $= \dfrac{\Delta l - \Delta l'}{l}$

$= \dfrac{\alpha \Delta T l - \Delta l'}{l}$

$= \alpha \Delta T - \dfrac{\Delta l'}{l}$

Thermal stress $=$ E $\times$ Thermal strain

$= E \times \left(\alpha \Delta T - \dfrac{\Delta l'}{l}\right)$

Simple problems on stress, strain and elastic constants

: Numericals :

Q.1) A Rod 150 cm long and of dia 2cm is subjected to an axial pull of 20 kN. If the modulus of elasticity of the material of the rod is to 2×10^5 N/mm². Determine (i) stress (ii) The strain (iii) The elongation of the rod.

Soln:- For rod

$l = 150$ cm

$d = 2$ cm, $r = 1$ cm

$F = 20$ kN $= 20000$ N

$E = 2\times10^5$ N/mm²

$= 2\times10^7$ N/cm²

$\Delta l = ?$, $\sigma = ?$, $\varepsilon = ?$

i) Stress $(\sigma) = \dfrac{F}{A} = \dfrac{20000}{\pi r^2} = \dfrac{2\times10^4}{3.14\times(1)^2} = \dfrac{2\times10^6}{314}$

$= 6369.42$ N/cm²

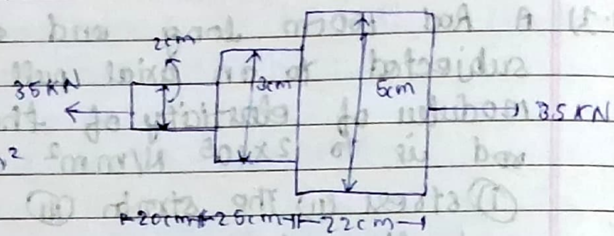
ii) Strain $(\varepsilon) = \dfrac{\sigma}{E} = 3184.71\times10^{-7}$

iii) Δl (Total elongation in the rod) $= \varepsilon \times l$

$= 3184.71\times10^{-7} \times 150$

$= 0.0477$ cm

Q.) An axial pull of 35000N is acting on a bar consisting of three lengths as shown in the fig. If the $E = 2.1\times10^5 N/mm^2$
i) σ in each section
ii) Δl of the bar



Sol^n:- Given,
$F = 35 KN$
$E = 2.1\times10^5 N/mm^2$

(I) For section 1
stress $(\sigma_1) = \frac{F}{A_1} = \frac{35000}{\pi r^2} = \frac{35\times10^3}{3.14\times(10)^2}$
$= 111.46 N/mm^2$

$E = \frac{\sigma_1}{e_1}$

$\Rightarrow e_1 = \frac{\sigma_1}{E}$

$\Rightarrow \frac{\Delta l_1}{l} = \frac{111.46}{2.1\times10^5}$

$\Rightarrow \frac{\Delta l_1}{200} = \frac{1114.6}{21\times10^5}$

$\Rightarrow \Delta l_1 = \frac{1114.6\times2}{21\times10^3}$
$= \frac{11146\times2}{21\times10^4}$
$= 0.10615 mm$

(II) For section 2
stress $(\sigma_2) = \frac{F}{A_2} = \frac{35000}{3.14\times(15)^2} = \frac{35000}{706.5} = 49.53 N/mm^2$

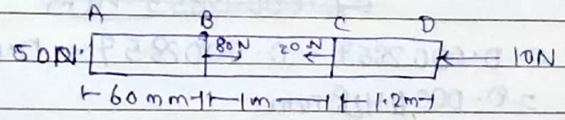
$\Delta l_2 = \frac{l_2\times\sigma_2}{E} = \frac{250\times49.53}{2.1\times10^5}$
$= \frac{12382.5}{21\times10^4}$
$= 0.0589 mm$

(III) for section 3
stress $(\sigma_3) = \frac{F}{A_3} = \frac{35000}{3.14\times(25)^2} = \frac{35000}{1962.5} = 17.83 N/mm^2$

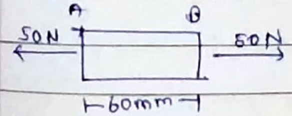
$\Delta l_3 = \frac{l_3\times\sigma_3}{E} = \frac{220\times17.83}{2.1\times10^5} = \frac{3922.6}{2.1\times10^5} = \frac{0.3922 mm}{2.1}$
$= 0.0186 mm$

Now,
$\Delta l = \Delta l_1 + \Delta l_2 + \Delta l_3$
$= (0.10615 + 0.0589 + \cancel{0.3922}\ 0.0186) mm$
$= \cancel{5.5}\ 0.1791 mm$

Q.) A Brass bar having cross-sectional area of $1000 mm^2$ is subjected to axial force as shown in the fig. Find the total elongation of the bar take $E = 1.05\times10^5 N/mm^2$



Sol^n:- For AB



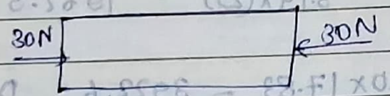
$\Delta l_1 = \frac{P_1\times l_1}{A\times E} = \frac{50\times60}{\cancel{3.14\times}\ 1.05\times10^5}$
$= \frac{3000}{\cancel{3.29\times10^5}\ 1000\times1.05\times10^5}$

$$\frac{3}{3.29\times10^2} = \frac{3}{329}$$

$$= \frac{3000}{105000000}$$

$$= 0.00002857\ mm$$

For BC



$$\delta l_2 = \frac{l_2 \times F_2}{A \times E} = \frac{1000\times 30}{1000\times 1.05\times10^5} = 0.0002857\ mm$$

For CD

$$\Delta l_3 = \frac{l_3 \times F_3}{A\times E} = \frac{1200\times 10}{1000\times 1.05\times10^5} = \frac{12}{1.05\times10^5} = \frac{11.42857}{1000000}$$

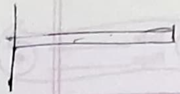
$$= 0.00011428$$

Total elongation $= \Delta l_1 + \Delta l_2 + \Delta l_3$

~~= 0.00042855 mm~~

$$= 0.0002857 - 0.0002857 + 0.00011428$$

$$= 0.00011428\ mm$$



CH- 7

→ Derivation of Bending eqn

→ Bending stress

→ moment of resistance

→ shear stress

7.) 1.) Pure Bending - Assumptions - Neutral Axis - Bending equation.

2.) Problems on Bending Eqn

(simple) Pure bending - also known as uniform bending

- Type of bending in ~~...~~ structural mechanics, where the bending occurs without any S.F or A.F.
- In this, a beam or structural member is subjected only to a bending moment, causing it to bend uniformly along its length.

Properties

1.) Constant Bending moment - The bending moment is constant throughout the length of the beam.

2.) Uniform Curvature: The beam is bent into a circular arc, and the curvature is uniform throughout the length of the beam.

3.) No Torsion: There is no torsion or twisting of the beam.

4.) No Axial force: There is no axial force of tension / ~~force~~ compression acting on the beam.

Importance

It is an important concept in structural mechanics and is used to analyze the behaviour of beams and other structural members under various loads.

- used to design bridges and buildings.

Assumptions

1) The material of the beam is homogeneous means the material is same over length.
2) " " " " " is ~~iso~~ isotropic means elastic prop. in all directions are equal.
3) The value of E is same in tension and compression.
4) The transverse sections which were plane before bending, remain plane after bending.
5) The radius of curvature is large as compared to the dim. of the cross-section.
6) Each layer of the beam is free to expand or contract.
7) Loading is within elastic limit.

Neutral axis

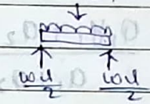
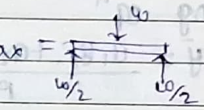
- imaginary line that runs through the cross-section of the beam, ⊥ to the plane of bending
- It is the line where the bending stress is 0.
- ~~It~~ separates the area of the beam that are in compression and tension.
- It passes through ~~cen~~ the centroid of the beam's cross-section.

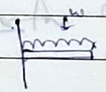
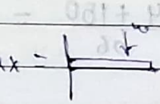
Bending Eq^n

- also known as the flexure formula, is used to calculate the bending stress in a beam.
- widely used in the design and analysis of beams and B.M.

$$\boxed{\frac{M}{I_{NA}} = \frac{\sigma}{y} = \frac{E}{R}}$$

where, M = Bending moment (Nm)
I_{NA} = Moment of inertia (m^4)
σ = Bending stress (N/m^2)
y = Distance from neutral axis (m)
E = Modulus of elasticity (N/m^2)
R = Radius of curvature (m)

$M_{max} = \frac{wl^2}{8}$  | $M_{max} = $  $= \frac{wl}{4}$

$M_{max} = \frac{wl^2}{2}$  | $M_{max} = $  $= wl$

Problems on Bending Eq^n

~~formula of centroid~~ for rectangle

$$\bar{x} = \frac{A_1x_1 + A_2x_2 \dots A_nx_n}{A_1 + A_2 \dots A_n}$$

$$\bar{y} = \frac{A_1y_1 + A_2y_2 \dots A_ny_n}{A_1 + A_2 \dots A_n}$$

$I_{xx} = \frac{bd^3}{12}$, $I_{yy} = \frac{db^3}{12}$

$I_{zz} = I_{xx} + I_{yy}$

$I_{NA} = \frac{\pi D^4}{64}$

for circle
$I = \frac{\pi D^4}{64}$

$ma = -kg \; m/s$

$\sigma_n = \sigma_x \cos^2\theta$

$\sigma_t = \frac{\sigma_x}{2} \sin 2\theta$

$\sigma_n = \left(\frac{\sigma_x+\sigma_y}{2}\right) + \left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\theta$

CH-8

FEM - Basic steps - Background of FEM & its need - numerical methods - Advantages - Disadvantages - Limitations - Appl^n of FEM, Concept of Discontinuity.

$\sigma_t = \left(\frac{\sigma_x-\sigma_y}{2}\right)\sin 2\theta$

CH-9

Phases of FEA (Finite Element Analysis)
2) Discretization process
3) Meshing - Element type

$\sigma_r = \sqrt{(\sigma_n)^2+(\sigma_t)^2}$

$\sigma_{n\,max} = \frac{\sigma_x}{2}$

$\sigma \cdot \left(\frac{\sigma_x-\sigma_y}{2}\right)$

CH-10

$\sigma_n = \sigma_x$

Stiffness Matrix of a Bar element
2) Global stiffness Matrix - properties of stiffness matrix
3) Boundary conditions - methods - types

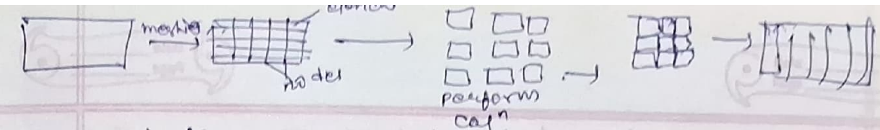
CH-11,12,13

Problems on 1-D, 2-D elements

CH-8

FEM - Finite Element Methods

FEM is the numerical technique used to perform FEA. It involves dividing a complex problem, like a structure or system, into smaller, simpler parts called "finite elements". It solves eq^n for each element, then combines the results to analyze the whole system.



Basic steps in FEM

1) Preprocessing (Modeling):
- Define the geometry of the problem.
- Discretize the model into smaller elements (mesh generation)
- Apply material prop. and boundary conditions.

2) Formulation:
- set up the mathematical eq^n for each element based on the physical behavior (e.g. stress, strain, heat).

3) Assembly:
- combine the element eq^n to create a global system that represents the entire structure.

4) Solution:
- solve the system of eq^n to obtain unknown values (like displacements, temp^n or forces).

5) Postprocessing:
- Analyze and visualize the results to interpret the behaviour of the entire system.

Background of FEM

1) 1940s - 1950s - FEM was developed for structural analysis in aerospace engg.

2) ~~1960~~ 1950: ~~Raymond~~ Clough W. Richard Courant ~~the~~ coined the term "Finite Element" in 1943 while working on aerospace projects.

3) 1970s - 1980s: FEM expanded to thermal, fluid and electromagnetic analysis, aided by improved computer technology.

4) 1990s - Present: FEM became widely accessible with the development of software like ANSYS and ABAQUS, and across industries like automotive, civil, and biomedical engg.

Numerical Methods

1) Finite Difference Method
2) " Vol. "
3) " Element "
4) Boundary " "
5) Meshless " "

Diff^n eq^n + BCs

↓

FEM

↓

Linear system of eq^n ($Ax=b$)

~~FEM~~ FEM

- Advantages
1) High acc. for complex problems.
2) Flexibility in analyzing various physical phenomena.
3) Handles complex geometries.
4) Provides detailed results.
5) Cost-effective and reduces physical testing.

- Disadvantages
1) Computationally intensive for large models.
2) Mesh quality impacts acc.
3) Setup can be complex and time-consuming.

4.) Require specialized knowledge
5.) Results are approximations.

Limitations
1.) Challenges with nonlinear problems.
2.) Accurate material models needed.
3.) Sensitive to boundary condition choices.
4.) Time-consuming for large simulations.

Applications
1.) Structural analysis (ex - buildings, bridges)
2.) Heat transfer (Ex - engine simulations)
3.) Fluid dynamics (Ex - airflow modeling).
4.) Biomechanics (ex - prosthetics)
5.) Automotive industry (Ex - crash testing).
6.) Electromagnetic simulations (ex - antenna design).

Concept of Discontinuity
A sudden change or break in a material/system's properties (e.g., stress, temp^n, displacement).

Types:
- Structural: Changes in geometry or material properties (eg. cracks)
- Material: Transition bet^n diff^n materials
- Displacement: Jump in displacement, such as at cracks
- Stress: Sudden change in stress, like at notches or crack tips.
- Thermal: Sudden change in temp^n or conductivity.

3.) Importance: can lead to stress concentrations, affecting strength and durability.

4.) FEM: Requires special handling in FEA to ensure acc.

CH — 09

1.) Phases of FEA (finite Element Analysis)
2.) Discretization process
3.) Meshing - Element type

- FEA is a simulation technique used to analyze how structures or systems behave under diff^n conditions.
- It divides complex st. into smaller elements for easier analysis
- Solve eq^n for each element to predict stress, deformation, heat transfer etc.
- used in engg. to optimize designs and reduce physical testing.
- Applied in industries like aerospace, automotive and civil engg.

Phases of FEA

1) Pre-processing:

- Modeling: define the geometry of the st./system.
- Meshing: divide the geometry into smaller, simpler elements.
- Mate. prop.:- Assign mate. prop. (ex - Young's modulus, density).
- Boundary conditions: Apply forces, constraints, or tempⁿ to the model.

2) Solving:

- ~~Formation~~ Formulation: Set up the system of eqⁿ based on element behaviour.
- Solving: Use numerical methods to solve the eqⁿ and determine unknown values like displacement, stress etc.

3) Post-processing:

- Results visualization: Interpret the results through graphs, contour plots, or animations.
- Analysis: Evaluate the performance, such as stress distribution, deformation, or thermal effects.
- Optimization: refine the design based on the results to improve performance or safety.

Discretization process

The discretization process in FEA involves breaking a continuous st. into smaller, simpler elements (like △s or ▭s) connected at nodes.

Key steps include:

1.) Geometry representation: define the st.'s shape as a mesh of elements.

2.) Meshing: divide the domain into small elements.

3.) Element selection: choose element types based on the problem's geometry.

4.) Degrees of freedom (DOF): Assign variables (like displacement) to each node.

5.) Approximation: use polynomial functions to represent continuous variables within elements.

Meshing - element type

~~In mes~~ Meshing in FEA is the process of dividing a st. into smaller, simpler elements (e.g., △s, ▭s or ⬚s) connected at nodes. It allows for numerical analysis of complex systems.

element type refers to the shape and d. of the individual comp. into which the model is divided.

Types

1) 1D elements (line elements):

- Beam " : used for analyzing st. under bending and axial loads.
- Truss el. - analyzing axial loads in st. like bridges & frames.

2) 2D el... (Plane el.)

- △ el.. : Used for irregular or curved geometries, commonly in 2D stress or heat transfer analysis.
- Quadrilateral el.. : Provide better acc. and are used in 2D analysis with rectangular or structured meshes.

3) 3D el... (solid elements):

- Tetrahedral el.... : Used for complex 3D geometries, especially when mesh is irregular.
- Hexahedral (Brick) el.. : Used for structured meshes, ideal for regular, box-shaped domains.

4) Shell el.. : Used for thin st. like plates or shells, where bending and stretching are imp.

5) Specialized el.. :-

- Fluid el.. - for fluid flow simulations.
- Axisymmetric el.. - used ~~F~~ when the geometry and loading are symmetrical around an axis.

1) Stiffness matrix of a bar element
2) Global stiffness matrix - properties of stiffness matrix
3) Boundary conditions - methods - Types

1) The stiffness matrix of a bar el. in FEA represents the relationship betn the forces applied at the nodes and the displacements in the system. For a simple 1D bar el. (like a truss or rod), the stiffness matrix relates the axial forces to the displacements along the length of the bar.
For a linear bar el. with constant cross-sectional area A, young's modulus E, and length L of the bar, the stiffness matrix [K] is given by:

$$[K] = \frac{EA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

~~dent~~ represents the relationship betn displacements at the two nodes.

Interpretation:

- ~~It~~ K is symmetric, meaning the force applied at one node affects the displacement of the other.
- The values 1 and -1 indicate the interaction betn the two nodes.

2. The global stiffness matrix is a square, symmetric matrix used in structural analysis, particularly in FEA, that represents the resistance of a st. to deformation. It relates forces and displacements at various degrees of freedom (DOF).

- It ensures stability and that energy is +.
- It's $\boxed{}$, with dim. corresponding to the no. of DOF,
- It often has many zero entries
- •

(G SM Assembly) $K_{global} = \sum_{e=1}^{n} T_e^T K_e T_e$

T_e = transformation matrix

n = no. of el.

Global system Eq^n:-

$$K_{global} u = F$$

u = vector of displacements

F = vector of external forces

3) Boundary conditions in FEA are constraints applied to a st. to simulate real-world interactions with its environment. They define how the st. is held, supported, or loaded, ~~and~~ .

Types:

(1) Displacement Boundary condition (Dirichlet boundary conditions) — These conditions specify the dis... or movement of certain points (nodes) in the st.

- To ~~purpo~~ fix or prescrible the displacement at
- specific locations.

- Ex -

•) Fixing a point in space so that it cannot move (ex- u=0, no displacement).

•) Specifying a known dis..., such as moving a point by 5mm in a specific direction.

(11) Force boundary conditions (Neumann Boundary conditions): - These conditions specify forces, tractions, or pr. applied to the st.

- To apply external loads that affect the st.'s behavior
- Ex -

• Applying a point load at a node (ex, F = 100N)

• " a pr. or distributed load to a surface or el...

Q) Why B.C are important

- realistic simulation, - unique solution, - determining reacti

Methods:

1) Direct Method (Penalty method): Increases stiffness at constrained DOFs to enforce boundary conditions.

2) Lagrange multiplier method: Introduces additional variables (multipliers) to enforce constraints while maintaining equilibrium.

3) Elimination method: Eliminates the constrained DOFs from the system and solves for the remaining unknowns.

4) Transformation method: Applies boundary conditions in a transformed reference frame and converts them back to global coordinates.

Basic steps of FEM

Pre-processing:

- Meshing - divide the domain into finite elements.
- Element selection - choose appropriate element types (e.g., beam, shell, solid).
- Material properties: Define modulus, Poisson's ratio, density, etc.
- Boundary conditions: Apply constraints (e.g., - fixed supports, displacements and loads.

Formulation

- Stiffness matrix: Derive local element stiffness matrices and assemble them into a global stiffness matrix
- Load vector: Formulate the global force vector, ~~vector~~ ~~loads~~, incorporating applied loads and boundary conditions.

Solution:

Solve system of Eq^n - use methods like Gaussian elimination or iterative solvers to solve for nodal displacements.

Post - processing:

Results Interpretation: Visualize displacements, stresses and strains using contour plot and deformed shapes.

Refinement

Mesh " - Refine the mesh or adjust parameters to improve accuracy and repeat the analysis if necessary.

Background of FEM

1) 1940s - Richard courant - introduced concept of finite element to solve differential eq^n in structural mechanics.

2) 1960s: Ray W. clough formalized FEM for structural analysis in his 1960 paper, marking its introduction as a key engg. tool.

3) 1970s: FEM expanded to fields like heat transfer and fluid dynamics. The first FEM software system were developed.

4) 1980s: With advancement in computing, FEM became widely used in industries like aerospace, automotive and civil engg. with commercial software like ANSYS and ABAQUS emerging

5) 1990s - Present - Increased computational power and software improvements made FEM a standard tool for complex simulations across various fields, including biomechanics

Concept of Discontinuity

Definition :- A discontinuity refers to the sudden changes in a structural material properties, geometry , boundary conditions and applied loads.

Types
1) Material d — changes in material properties. (eg. steel to concrete).

2) Geometric d — changes in shape or structure. (eg. notches, holes).

3) Boundary d — changes in support or constraints (eg. fixed to simply supported)

4) Load Discontinuity : sudden changes in applied loads (eg. point loads)

Effects
- Leads to the stress concentrations and potential material failure.
- Causes stress redistribution in structure.

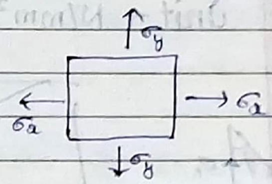
Analysis : Discontinuities make analysis complex, often requiring FEM for accurate modelling and predictions.

Importance
Understanding & addressing discontinuities is crucial for ensuring safety and stability of structures.

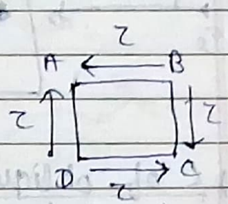
Example :-
Notch in a beam (in ~~mid~~ midspan)

Principal stresses & planes

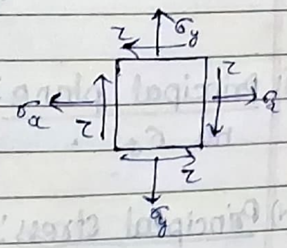
(i) A body or a plane may be subjected to only normal stress (either tensile or compressive stress).



(ii) A body or a plane may be subjected to only shear stress (Tangential stress)



(iii) A body or a plane may even be subjected ~~of~~ to a combination of normal stress as well as tangential stress (shear stress)

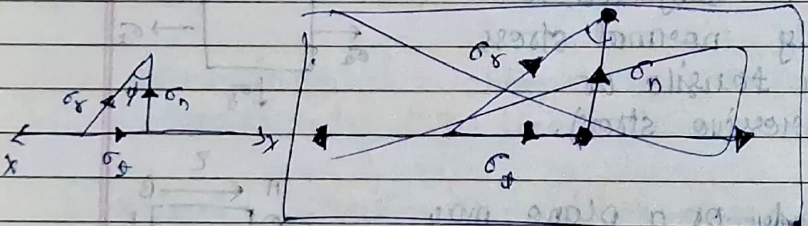


⇒ Tensile or compressive stress given in the problem would be denoted by 'σ_x' or 'σ_y' or vice versa.

⇒ Tangential stress will be denoted by 'σ_t'.

* Some imp. definitions in principal stresses & planes :

(i) Resultant stress (σ_r) :- resultant of normal stress and ~~tan~~ σ_t.
unit - N/mm^2, N/cm^2, N/m^2.



(ii) ∠ of obliquity :- ∠ made by the σ_r with σ_n, denoted by ϕ

$$\tan\phi = \frac{\sigma_t}{\sigma_n} \quad \text{or} \quad \phi = \tan^{-1}\left(\frac{\sigma_t}{\sigma_n}\right)$$

(iii) Principal plane :- plane which carries only σ_n & no σ_t.

(iv) Principal stress :- The value or magnitude of a σ acting on a principal plane.

(v) Major principal ~~plane~~ stress :- The max^m value of σ_n acting on a principal plane.

(vi) Minor principal ~~plane~~ stress :- The min^m value of σ_n acting on a principal plane.

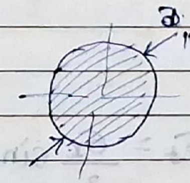
(vii) Major principal plane - plane which carries max^m σ_n.

(viii) Minor principal plane - plane which carries min^m σ_n.

Q. A 12mm dia bar is subjected to a pull of 12 kN. Calculate the normal & tangential stresses on planes making ∠s of 5° & 37° with the axis of the bar.

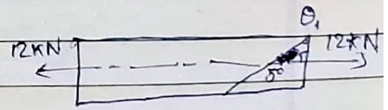
Sol^n :- Given data :

D = 12 mm
P = 12 kN = 12×10^{-3} N
σ_n = ? θ = 5° & 37°
σ_t = ?



cross-sectional area of the bar :-

$$\frac{\pi}{4}(D)^2 = \frac{22}{7} \times \frac{(12)^2}{4} = 113.142\ mm^2$$



Case No. 1

$\theta_0 = 5°$
$\theta_1 = 90 - 5 = 85°$

Stress along x-direction :-

$$\boxed{\sigma_x = \frac{P}{A}} \Rightarrow \sigma_x = \frac{12 \times 10^3}{113.142} = 106.061\ N/mm^2$$

So, $F_1 \cos\theta = F_3$

$F_1 \sin\theta = F_2$

From eq (i) & (ii)

$$\boxed{P_1 = P_2 = P_3}$$

Force

Force all 3 laws

Force may be defined as an action which changes or tends to change the state of rest or of uniform motion of a body. For representing the force acting on a body, the magnitude of a force, point of action and direction of its action should be known.

A.T newton's 2nd law of motion, force can write as

$F = ma$

$= \text{mass} \times \text{acce}^n$

$= \text{mass} \times \frac{\text{length}}{\text{Time}^2}$

One Newton force is defined as the amount of force required to accelerate a mass of 1 kg by $1\ m/s^2$.

Thus,

$$1\ N = 1\ kg \times 1\ m/s^2 = 1\ kg\text{-}m/s^2$$

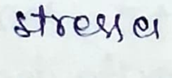
Effects of a force :- A force produce following effects in a body, on which it acts :

1) It may change the state of motion of a body i.e. If a body is at rest, then the force may set it in motion And if a body is already in motion, then the force may accelerate or slow down it.
2) It may accelerate or retard the motion of a body.
3) A force can change the direction of a moving object without changing its speed (e.g. turning a car).
4) It may deform the body on which it acts and produce internal stresses and strains.

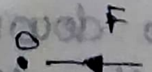
characteristics of a force :-

1) Magnitude: The quantity of force (10N, 50kN, 5 tonnes etc.)
2) Direction: The direction of ~~a long~~ the line along which the force acts. It is also known as line of action of the force.
3) Point of application: The point at which the force acts on the body.
4) Nature : (i) Pull or (ii) Push

(i) Pull : If the arrow head is pointed away from the point of appl^n, then the nature of the force is pull.



(ii) Push: If the arrow head is pointed towards the point of appl^n, then the nature of the force is push.



16 # Force can be divided into primarily into two types of forces:-

1.) Contact forces   2.) Non-contact forces

• Contact forces – Those forces ~~that~~ occur only when two objects are in physical contact with each other. The force is tramistted through touch. All the mech. forces are contact forces. Some of the examples of contact forces in day to day life are – pushing a door, playing basketbal, Hitting the football, working on laptop, pushing sofa and so on.

Types of contact forces:-

1.) Spring force: force exerted by a compressed or stretched spring is spring force. The force created could be pull or push depending on how the spring is attached.

2.) Applied force: Any force that is applied to an object by another object or person.
Ex – Pushing a door

3.) Air resistance force:- (Drag force) – type of frictional force acting against the motion of an object moving through air. It increases with speed. Ex – A skydriver slowing down using a parachute.

4.) Frictional force:- A force that opposes the motion of an object when it moves or tries to move on a surface. It can slow down or stop moving objects. Ex – A ball rolling in a ground eventually stops due to friction.

5.) Normal force – The support force exerted by the surface to support the weight of the body resting on it. Acts ⊥ to the surface.
Ex – A book lying on a table.

6.) Tensional force – The force that is produced with the help of rope, string, cable or wire. It doesn't have pushing ability.
Ex – pulling a box with a rope.

• Non-contact forces – force that acts on an object without any physical contact betn the body and the source of force. These forces act from a dist. through invisible fields.

1) Gravitational force – force of attraction betn two objects having mass. It always ~~push~~ pulls the objects towards each other, never repels.
Ex – Earth pulling a apple down when it falls.

2) Magnetic force – force exerted by magnets. It can attract magnetic materials like iron, cobalt and nickel. It can repel or attract other magnets depending on their poles. Like poles repel, unlike poles attract.

3) Electrostatic force – The force betn objects that are electrically charged. Opposite charges attract and like charges repel.
Ex – Rubbing a balloon to attract paper.

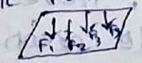
Force system

Coplanar forces
→ collinear
→ concurrent
→ non-concurrent
→ parallel forces < Like, unlike

Non-coplanar forces
→ N.c concurrent F.S
→ N.c ∥ F.S
→ N.c Non-concurrent F.S

A force system is a collection of forces acting on a body or object. These forces can vary in direction, magnitude and point of appl^n.

Coplanar force system – when all the force lie in a single plane then that force system is called coplanar force. Common in 2D static problems



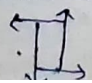
Non-coplanar force system – when all the force lie in a diff^n planes (3D). More complex, used in spatial analysis

1) Coplanar force types

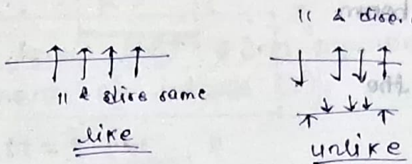
* Collinear force system – when all the forces in a line.

* Concurrent force system – when all the forces passing through a single point.

* Non-Concurrent force system – when all the forces passing through diff^n points.



* ∥ force system – when all the forces are ∥ to each other. Type – like & unlike



Relation bet^n K, G and E

• Relation bet^n E and G

$$E = 2G(1+\mu)$$

$$\Rightarrow \frac{E}{2G} = 1+\mu$$

$$\Rightarrow \boxed{\mu = \frac{E}{2G} - 1} \quad \text{——— (1)}$$

• Relation bet^n E and K

$$E = 3K(1-2\mu)$$

Putting eq (1)

$$E = 3K\left\{1 - 2\left(\frac{E}{2G} - 1\right)\right\}$$

$$\Rightarrow E = 3K\left(1 - \frac{2E}{2G} + 2\right)$$

$$\Rightarrow E = 3K\left(3 - \frac{2E}{2G}\right)$$

$$\Rightarrow E = 3K\left(\frac{3G - E}{G}\right)$$

$$\Rightarrow EG = 9KG - 3KE$$

$$\Rightarrow EG + 3KE = 9KG$$

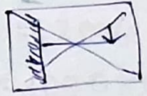
$$\Rightarrow E(G+3K) = 9KG$$

$$\Rightarrow \boxed{E = \frac{9KG}{3K+G}}$$

MOM

S.F & B.M

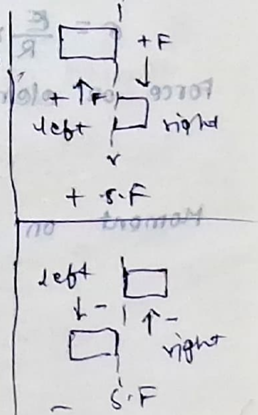
Point of contraflexure – point at which no bending occur. Or it is the point ~~where~~ at which bending moment curve intersect with zero line. Where the bending moment changes its sign from + to – or vice versa.



Beam – A st. member subjected to a external forces at the right ∠s to the longitudinal axis. It is vertically loaded.

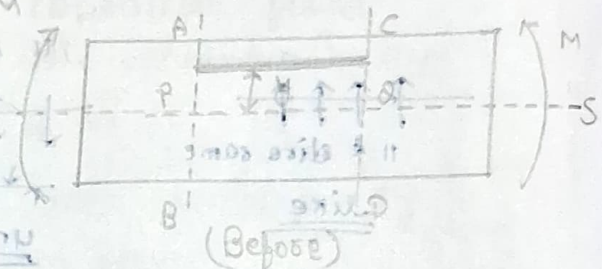
# Simply supported beam
S.F = + and –
B.M = +

# Cantilever beam
S.F = +
B.M = –



Bending Eq^n

Derivation:-

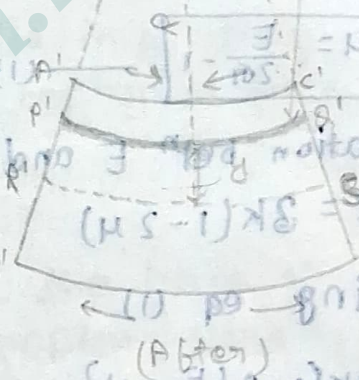


Let, M = Moment acting at the beam
$\theta = \angle$ of Bending
R = Radius of curvature of the beam

$$\text{Strain}(e) = \frac{\text{Change in length}(\Delta l)}{\text{Original length}(l)}$$

$$= \frac{PQ - P'Q'}{PQ} \qquad \left[\begin{array}{l} P'Q' = (R-y)\theta \\ PQ = R\theta \end{array}\right]$$

$$= \frac{R\theta - (R-y)\theta}{R\theta}$$



$$= \frac{R\theta - R\theta + y\theta}{R\theta}$$

$$e = \frac{y\theta}{R\theta}$$

$$\boxed{\frac{\sigma}{E} = \frac{y}{R}} \qquad \left[\because e = \frac{\sigma}{E}\right]$$

$$\boxed{\frac{\sigma}{y} = \frac{E}{R}} \quad -(1)$$

$$\sigma = \frac{E}{R} \times y$$

Force on elementry strip $= \sigma . da$ (Area = da)

$$= \frac{E}{R} \times y \times da$$

Moment on elementry strip = Force $\times \perp$ distance

$$dM = \frac{E}{R} \times y \times da \times y$$

$$dM = \frac{E}{R} \times y^2 \times da$$

Taking both side integration

$$M = \frac{E}{R} \int y^2 . da$$

We know that 2nd moment of area is called Moment of interia (I).

$$M = \frac{E}{R} . I$$

$$\Rightarrow \frac{M}{I} = \frac{E}{R} \quad \text{---} (II)$$

From eq (I) & (II)

$$\boxed{\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}} \quad \text{Bending eq}^n$$