EPE

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UPA

CH - 02

(i) Explain Discharge through rectangular Notch - Numerical.
(ii) Explain Discharge through V-Notch - Numerical.
(iii) Explain flow through pipes - major and minor losses.

* Notch :- A notch is device used for measuring the rate of flow of a liquid through a small channel or a tank.
• It may be defined as an opening in the side of a tank or a small channel in the side of a tank or a small channel in such a way that the liquid surface in the tank or channel is below the top edge of the opening. [The bottom end of the notch is generally made sharp "made up of a metal plate used to regulate fluid flow at or .]

* Weir :- A weir is a conrete or masonry structure, placed in an open channel over which the flow occurs.

• It is generally in the form ~~of a~~, wall with a sharp edge at the top.
• The notch is of small size while the weir is of bigger size.
• The notch is generally made of metallic plate while weir is made of concrete or masonary structure.

* Nappe or vein :- The sheet of water flowing through a notch or over a weir.

* Crest or sill :- The bottom edge of a notch or a top of a weir over which the water flows.

Rectangular notch - It is a common type of weir used to measure the flow rate of liquids. It consists of a vertical opening with a rectangular shape, often placed in a channel or tank.

→ Easy to construct and maintain.

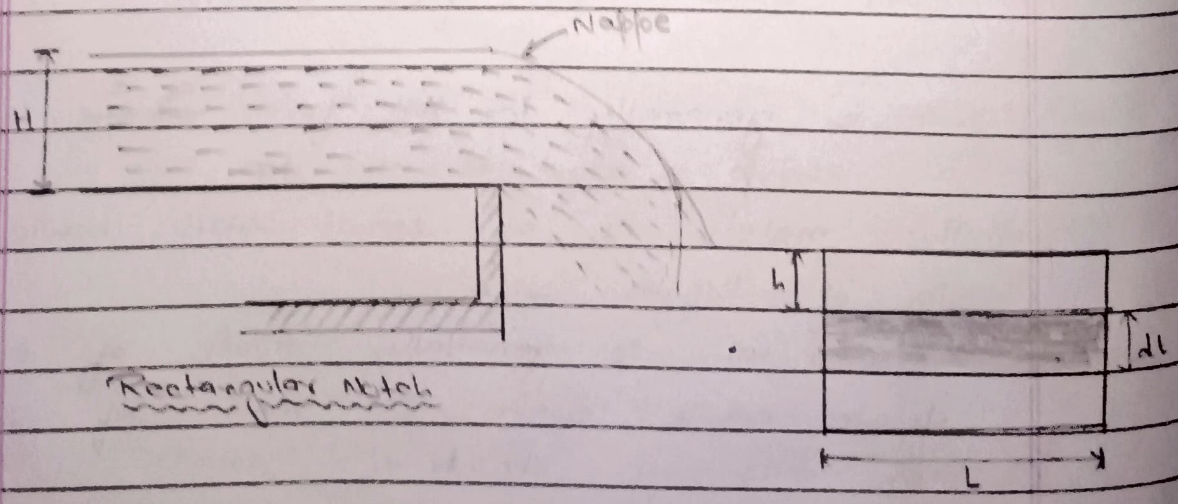
→ Provides reliable flow rate readings.

→ Suitable for various flow rates and liquid types.

→ used in hydrology, industrial processes, research and development.

→ Sharp, Broad - crested R.N.

* Discharge through Rectangular notch :-



Rectangular Notch

The expression for discharge over a notch and weir is the same.

Consider,

a rectangular notch provided in a channel carrying water.

Let,

H = Head of water over the crest

L = length of the notch

consider,

An elementary horizontal strip of water of thickness dH and length L at a depth h from the free surface of water.

Area of strip = L × dh

Theoretical velocity of water flowing through strip

$= \sqrt{2gh}$

The discharge dQ through strip is,

$dQ = cd \times \text{Area of Strip} \times \text{Theoretical velocity}$

$= cd \times L \times dh \times \sqrt{2gh}$

where,

cd = co-efficient of discharge

The total discharge for whole notch is –

$$Q = \int_0^H cd \cdot L \cdot \sqrt{2gh} \cdot dh$$

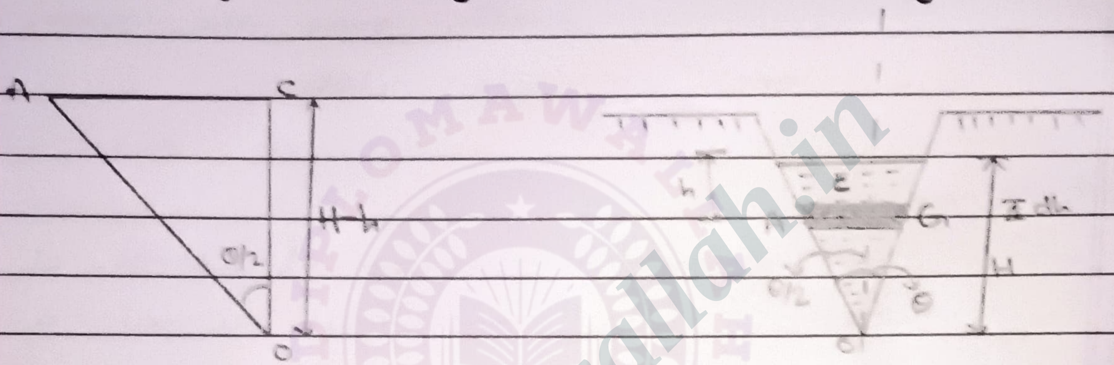
$$= cd \cdot L \times \sqrt{2g} \int_0^H h^{1/2}\, dh$$

$$= cd \cdot \times L \times \sqrt{2g} \left[\frac{h^{3/2}}{3/2}\right]_0^H$$

$$= \frac{2}{3} c_d \times L \times \sqrt{2g}\,[H]^{3/2}$$

$$\boxed{Q = \frac{2}{3} c_d \times L \times \sqrt{2g}\,[H]^{3/2}}$$ → Discharge through rectangular notch

* Discharge through v-notch (triangular notch) :-



V-notch

The expression for the discharge over a v-notch and weir is the same.

Let,

H = Head of water above the v-notch

θ = ∠ of the notch

Consider,

A horizontal strip of water thickness 'dh' at a depth of h from the free surface of water.

Then, In △ACO

$$\tan\frac{\theta}{2} = \frac{AC}{OC} = \frac{AC}{(H-h)}$$

$$AC = (H-h)\tan\theta/2$$

* width of notch at the water surface = $2H\tan\frac{\theta}{2}$

width of the strip = AB = 2AC

$= 2(H-h)\tan\theta/2$

So,

Area of strip $= 2(H-h)\tan\frac{\theta}{2} \times dh$

The theoretical velocity of water through strip
$= \sqrt{2gh}$

$\therefore$ Discharge through the strip

$$dQ = C_d \times \text{area of strip} \times \text{velocity}$$
$$= C_d \times 2(H-h)\tan\theta/2 \times dh \times \sqrt{2gh}$$

Total discharge,

$$Q = \int_0^H 2C_d(H-h)\tan\frac{\theta}{2} \times \sqrt{2gh} \times dh$$

$$= 2C_d \times \tan\frac{\theta}{2} \times \sqrt{2g}\int_0^H (H-h)\,h^{1/2}\,dh$$

$$= 2C_d \times \tan\frac{\theta}{2} \times \sqrt{2g}\int_0^H \left(Hh^{1/2} - h^{3/2}\right)dh$$

$$= 2C_d \times \tan\frac{\theta}{2} \times \sqrt{2g}\left[\frac{Hh^{3/2}}{3/2} - \frac{h^{5/2}}{5/2}\right]_0^H$$

$$= 2C_d \times \tan\frac{\theta}{2} \times \sqrt{2g}\left[\frac{H-H^{3/2}}{3/2} - \frac{H^{5/2}}{5/2}\right]$$

$$= 2C_d \times \tan\frac{\theta}{2} \times \sqrt{2g}\left[\frac{2}{3}H^{5/2} - \frac{2}{5}H^{5/2}\right]$$

$$= \frac{8}{15}C_d \times \tan\frac{\theta}{2} \times \sqrt{2g} \times H^{5/2}$$

$$\boxed{Q = \frac{8}{15}C_d \times \tan\frac{\theta}{2} \times \sqrt{2g}\,[H]^{5/2}}$$ $\longrightarrow$ discharge through v-notch

For a right angled v notch, if $cd = 0.6$

$$\theta = 90^\circ$$

$$\tan\theta = 1.$$

Then Discharge,

$$Q = \frac{8}{15} \times 0.6 \times 1 \times \sqrt{2 \times 9.81} \times H^{5/2}$$

$$\boxed{Q = 1.417\ H^{5/2}}$$

Explain flow through pipes:-

Major and minor ~~substance~~ losses when a fluid is flowing through a pipe. The fluid experiences some resistance due to which some of the energy of fluid is lost. This loss of energy is classified as:-

Energy losses

(i) Major energy losses
- This is due to friction
- It is calculated by:-
(i) Darcy-weisbach formula.
(ii) Chezy's formula

2) Minor energy losses
- This is due to:-
(i) sudden expansion of pipe
(ii) sudden contraction of pipe
(iii) Bend in pipe
(iv) Pipe fittingetc
(v) An obstruction in pipe

(1) Major energy losses:- Major losses occurs due to the friction betn the fluid and the pipe wall over the length of the pipe. and it is calculated by the following formula:-

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(a) Darcy - weisbach formula : The loss of head (or energy) in pipes due to friction is calculated from Darcy - weisbach eqn.

$$\boxed{h_f = \frac{4fLV^2}{d \times 2g}}$$

where, h_f = loss of head due to friction

f = co-efficient of friction which is function of reynolds no. (Re).

$= \frac{64}{Re}$ for $Re < 2000$ (laminar or viscous flow)

$= \frac{0.079}{Re^{1/4}}$ for Re varying from 4000 to 10^6

L = Length of pipe

v = mean velocity of flow

D = Diameter of pipe

(b) Chezy's formula for loss of head due to friction in pipes :-

$$h_f = \frac{f'}{\rho g} \times \frac{P}{A} \times L \times v^2 \quad \text{---- (i)}$$

where,

h_f = loss of head due to friction

P = wetted perimeter of pipe

A = Area of cross - section of pipe

L = length of pipe

v = mean velocity of flow.

The ratio of A to P is called hydraulic mean depth or hydraulic radius and is denoted by m.

$\therefore$ hydraulic mean depth $(m) = \frac{A}{P} = \frac{\pi/4 d^2}{\pi d}$

$= \frac{d}{4}$

$m = \frac{A}{P}$, So, $\frac{P}{A} = \frac{1}{m}$

Substituting the value of P/A in eqn (i)

$$h_f = \frac{f'}{\rho g} \times \frac{1}{m} \times L \times v^2$$

$$v^2 = h_f \times \frac{\rho g}{f_1} \times m \times \frac{1}{L}$$

$$= \frac{\rho g}{f_1} \times m \times \frac{h_f}{L}$$

$$v = \sqrt{\frac{\rho g}{f_1} \times m \times \frac{h_f}{L}}$$

$$v = \sqrt{\frac{\rho g}{f'}} \times \sqrt{\frac{m h_f}{L}} \quad \text{—— (ii)}$$

Let,

$\sqrt{\frac{\rho g}{f'}} = C$, where C is a constant known as chezy's constant

$\frac{h_f}{L} = i$, where i is loss of head per unit of pipe

Substituting the value of $\sqrt{\frac{fg}{f'}}$ and $\frac{h_f}{L}$ in eqn (ii)

$$\boxed{V = C\sqrt{mi}}$$

→ chezy's formula

The value of m for pipe is always equal to d/4.

(2) Minor Energy (Head) losses:-

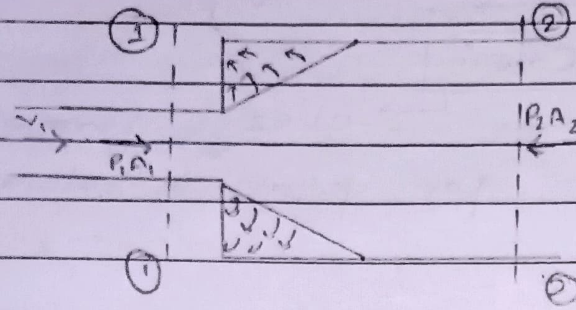
The loss of head or energy due to friction in a pipe is known as major loss while the loss of energy due to change of velocity of fluid in magnitude or direction.

The major loss of energy includes following cases:-

(i) Loss of energy due to sudden enlargement.
(ii) " " " " " sudden contraction.
(iii) " at the entrace of a pipe.
(iv) " " " exit of a pipe.
(v) " " due to obstruction in a pipe.
(vi) " due to bend in the pipe.
(vii) " in various pipe fittings.

- In case of long pipe the above looses are small as compared with the loss due to friction.
- In case of a short pipe, these losses are comparable with the loss of head due to friction.

① loss of head due to sudden enlargement :-



Let,

P_1 = Pressure intensity at section 1-1

v_1 = velocity of flow at section 1-1.

A_1 = Area of pipe at section 1-1.

Similary,

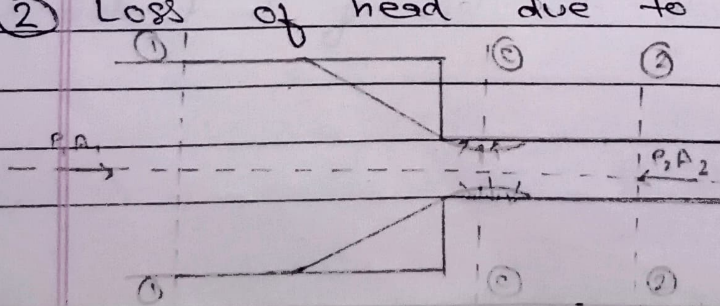
P_2 v_2 & A_2 are pressure, velocity & area at section 2-2.

- Due to sudden change of diameter of the pipe from D_1 to D_2 the liquid following from the smaller pipe is not able to flow the abrupt change of the boundary.
- Thus the flow separates from the ~~abrupt~~ boundary and turbulent eddies are formed.
- The loss of head or energy takes place due to the formation of these eddies.

$$\boxed{h_e = \frac{(v_1 - v_2)^2}{2g}}$$

where, h_e = loss of head due to sudden enlargement

② Loss of head due to sudden contraction :-



Sudden contraction

The liquids flows from large pipe to smaller pipe and the area of flow goes on decreasing and the area of flow goes on decreasing and becomes minimum at a section c-c.
This section c-c is called vena-contracta.
After section c-c, a sudden enlargement of the area takes place.
The loss of head due to sudden contraction is actually due to sudden enlargement from vena-contracta to smaller pipe.

$$h_c = \frac{kV_2^2}{2g} \qquad \text{where, } k = \left[\frac{1}{C_c} - 1\right]^2$$

if $C_c = 0.62$

$$k = \left[\frac{1}{0.62} - 1\right]^2 = 0.375$$

then,

$$h_c = \frac{kV_2^2}{2g} = \frac{0.375\,V_2^2}{2g}$$

If the value of C_c is not given

$$\boxed{h_c = 0.5\,\frac{V_2^2}{2g}}$$

(3) Loss of energy at the entrance to pipe.

$$\boxed{h_i = 0.5\,\frac{V^2}{2g}}$$

where, V = velocity of liquid in pipe
h_i = loss of head at entrance

(4) Loss of energy at the exit from of pipe

$$h_o = \frac{V^2}{2g}$$ h_o = loss of head at the exit

V = velocity at outlet of pipe

(5) Loss of energy due to obstruction in pipe

$$h_o = \frac{k (V_1 - V_2)^2}{2g}$$

(6) Loss of energy in bends,

$$h_b = \frac{kV^2}{2g}$$

h_b = loss of head due to bends

V = velocity of flow

K = Co-efficient of bend

(7) Loss of energy in various pipe fittings

$$h_e = \frac{KV^2}{2g}$$ where, V = velocity of flow

K = Co-efficient of pipe fitting

Numerical :- based on Rectangular Notch

(1) A rectangular notch 1 m wide has a constant head of 30mm. Find the discharge over the notch, if cd = 0.62

Soln :- given : for rectangular Notch
$L = 2m$, $H = 300\,mm = 0.3\,m$, $cd = 0.62$

By using formula :-

$$Q = \frac{2}{3}\, cd\, L \sqrt{2g}\; H^{3/2}$$

$$Q = \frac{2}{3} \times 0.62 \times 2 \sqrt{2 \times 9.8} \times (0.3)^{3/2}$$

$$\boxed{Q = 0.6016\ m^3/s}$$

Ans

(2) In a rectangular notch, discharge of water is $2\,m^3/s$ if the length of the notch is 6m, determine the head above the sill, Assume cd = 0.62

Soln :- given, rectangular, $Q = 2\,m^3/s$, $L =$
$cd = 0.62$, $H = ?$

$$Q = \frac{2}{3}\, cd\, L \sqrt{2g}\; H^{3/2}$$

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Numerical based on V-Notch :-

1. A 90° V-notch is used to measure the discharge of water. The depth of water above 30 cm. calculate the discharge in lit/sec. cd=0.62

Soln: given, $\theta = 90°$, $H = 30\,cm = 0.30\,m$, $cd = 0.62$, $Q = ?$

$$Q = \frac{8}{15} cd \sqrt{2g} \tan\left(\frac{\theta}{2}\right) H^{5/2}$$

$$= \frac{8}{15} \times 0.62 \times \sqrt{2 \times 9.81} \tan 45° \times (0.30)^{5/2}$$

$$= 0.33 \times 4.43 \times 1 \times 0.0493$$

$$= 0.072\ m^3/sec$$

$$= 0.072 \times 1000$$

$$= 72\ lit/sec$$ Ans

2. Find the discharge over a triangular notch of ∠s 120° when the head over the triangular notch 25 cm. Take cd = 0.62

Soln: given, $\theta = 120°$, $H = 25\,cm = 0.25\,m$, $cd = 0.62$
$Q = ?$

$$Q = \frac{8}{15} \times cd \sqrt{2g} \tan\left(\frac{\theta}{2}\right) H^{5/2}$$

$$= \frac{8}{15} \times 0.62 \times \sqrt{2 \times 9.81} \times \tan 60° \times (0.52)^{5/2}$$

$$Q = 0.799\ m^3/s$$ Ans

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3. Calculate the ∠s of a V-notch which discharge 0.0375 m^3/s of water under a head of 0.3m take $cd = 0.62$?

Soln:- $Q = 0.0375\ m^3/s$, $H = 0.3m$, $cd = 0.6$, $\theta = ?$

$$Q = \frac{8}{15}\, cd \tan\left(\frac{\theta}{2}\right)\sqrt{2g} \times H^{3/2}$$

$$0.0375 = \frac{8}{15} \times 0.6\sqrt{2\times 9.81} \times 0.3^{5/2} \times \tan\frac{\theta}{2}$$

$$\tan\left(\frac{\theta}{2}\right) = \frac{0.0357}{0.0699} = 0.536$$

$$\frac{\theta}{2} = \tan^{-1}(0.536)$$

$$\frac{\theta}{2} = 28.6 \quad , \frac{\theta}{2} = 28.6\times 2$$

$$\boxed{\theta = 56.4^\circ}$$

4. During an experiment in a laboratory, 26 l/s of water was found to be discharged over a right angled V-notch under a head of 200mm. Determine the co-efficient of discharged for the notch.

Soln:- given, $\theta = 90^\circ$, $Q = 26\ lit/s = \frac{26}{1000} = 0.026\ m^3/s$

$H = 200mm = 0.2m$, $cd = ?$

$$Q = \frac{8}{15}\, cd \tan\left(\frac{\theta}{2}\right)\sqrt{2g}\ H^{5/2}$$

$$0.025 = \frac{8}{15}\, cd \tan 45^\circ \sqrt{2\times 9.81} \times (0.2)^{5/2}$$

$$cd = \frac{0.026}{0.0422} = 0.62 \Rightarrow \boxed{cd = 0.62}$$

Assumptions of Darcy & chezy's eqn:

Steady flow: the flow parameters (velocity, pressure) do not change

Incompressible fluid:- density is constant

uniform cross section: the pipe diameter remains constant throughout the length.

fully ~~depend~~ developed flow:- velocity profile is fully developed & doesn't change along the length

~~uniform cross section~~:

single phase flow:- No mixture of gases/liquids/solid.

Flow can be laminar & or turbulent

Head loss only due to friction: No losses from fittings, bends, or valves in the base formula.

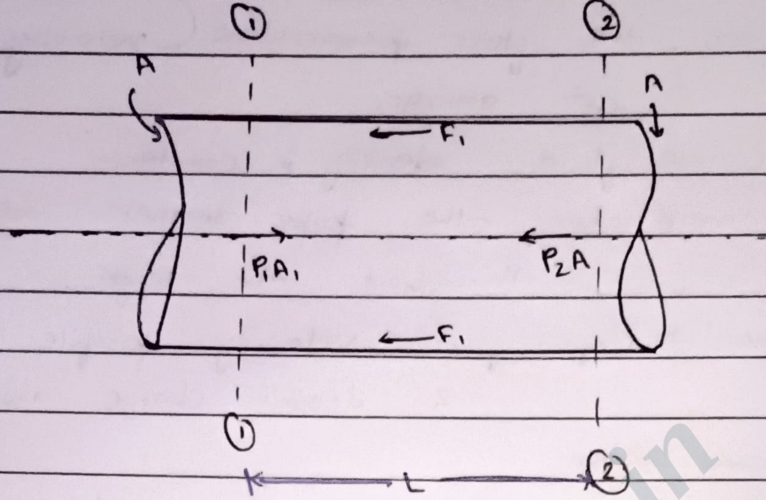
Chazy's Assumptions:

1. steady flow:
2. uniform flow:
3. Incompressible fluid:
4. Prismatic channel:

one dimensional motion.

Applies to open chanel:

Expression for loss of Head Due to friction pipe



Let,

P_1 = pressure intensity at inlet (1-1)

P_2 = pressure intensity at outlet (2-2)

A = Cross - section area of the pipe

L = length of Pipe

we know that,

[Bernoulli eq]

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + Z_2 + h_L$$

friction Loss

$\because Z_1 = Z_2$ (Head is costant)

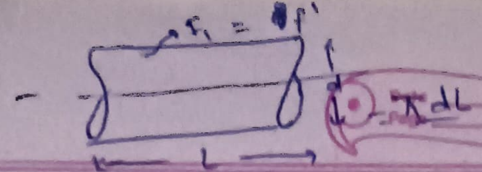
$v_1 = v_2$ (

[$Q = a_1 v_1 = a_2 v_2$ Continuity]

$$= \frac{P_1}{\rho g} = \frac{P_2}{\rho g} + h_L$$

$$\boxed{h_L = \frac{P_1}{\rho g} - \frac{P_2}{\rho g}}$$

$$= \boxed{P_1 - P_2 = \rho g h_L} \quad -1$$



Friction force = friction resistance per wetted area × velocity 2.

$f_1 = f' \times \pi dL \times v^2$

$\boxed{f_1 = f' \times P \times L \times v^2}$ $\left\{ \begin{matrix} \text{circular body} \\ P = \pi d \end{matrix} \right\}$ Perimeter

$\Sigma fx = 0$

$P_1 A - P_2 A - f_1 = 0$

$\Rightarrow (P_1 - P_2) A = f_1$

$\Rightarrow P_1 - P_2 = \frac{f' \times P \times L \times v^2}{A}$

$\Rightarrow \rho g h_L = \frac{f' \times P \times L \times v^2}{A}$

$\Rightarrow h_L = \frac{f' \times P \times L \times v^2}{\rho g \times A}$ $\left[\frac{P}{A} = \frac{\pi d}{\pi d^2} \times 4 = \frac{4}{d} \right]$

$\Rightarrow h_L = \frac{f'}{\rho g} \times \frac{4}{d} \times L \times v^2$

$\Rightarrow \boxed{\frac{f'}{\rho} = \frac{f}{2}}$ — Darcy-criteria

$\Rightarrow h_L = \frac{f \times 4 \times L \times v^2}{2 \times d \times g}$

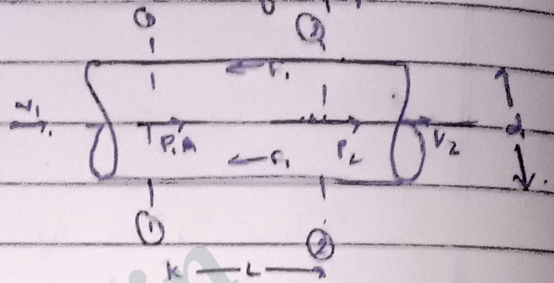
$\Rightarrow \boxed{h_L = \frac{4 f L v^2}{d \times 2g}}$ ← Darcy eqn for friction loss.

Head Loss due to friction

Derivation of Darcy - weisbach eqⁿ.

Consider a uniform Horizontal pipe, having stead flow.

Let 1-1 & 2-2 are two section of pipe.



P_1 = Pressure intensity at section 1-1.

P_2 = e.s.

A = area of pipe

L = length of pipe

v_1 = velocity of flow at section 1-1.

v_2 = " " " " 2-2.

F = friction resistance per unit wetted area per unit velocity^2.

h_f = loss of head due to friction.

Applying bernoulli's eqⁿ between section 1-1 & 2-2.

Total head at 1-1 = Total head at 2-2 + loss of head due to friction between 1-1 & 2-2.

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + h_f$$

friction 1-2

But, $z_1 = z_2$ as pipe is horizontal.

$v_1 = v_2$ as velocity of pipe is same between 1-1 & 2-2.

$$\therefore \boxed{\frac{P_1}{\rho g} = \frac{P_2}{\rho g} + h_f}$$

$$\frac{P_1}{\rho g} - \frac{P_2}{\rho g} = h_f$$

$$\boxed{(P_1 - P_2) = h_f \rho g} \quad — (i)$$

Friction resistance $= f' \times$ wetted area $\times$ velocity^2

$= f' \times \pi d L \times V^2$

$$\boxed{F = f' \times P \times L \times V^2} \quad — (ii)$$

$[\because \pi d = \text{perimeter}]$

The force acting on the fluid between 1-1 & 2-2 are:

(i) at section 1 – 1,

$$P_1 = \frac{F_1}{A_1}$$

Pressure force, $F_1 = P_1 A$ — (+)

(ii) at section 2-2,

$$P_2 = \frac{F_2}{A}$$

$$F_2 = P_2 A \quad — (-)$$

(iii) Frictional force, F — (−)

Resolving all the horizontal forces,

$$P_1 A - P_2 A - F = 0$$

$$P_1 A - P_2 A = F$$

$$(P_1 - P_2) A = F$$

$$P_1 - P_2 = \frac{F}{A}$$

$$h_f \rho g = \frac{f' \times P \times L \times V^2}{A}$$

$$h_f = \frac{f'}{\rho g} \times \frac{P \times L \times V^2}{A}$$

$$h_f = \frac{f'}{\rho g} \times \frac{4}{d} \times L \times V^2$$

$\left\{\because \frac{P}{A} = \frac{\pi d}{\pi d^2} \times 4 = \frac{4}{d}\right\}$

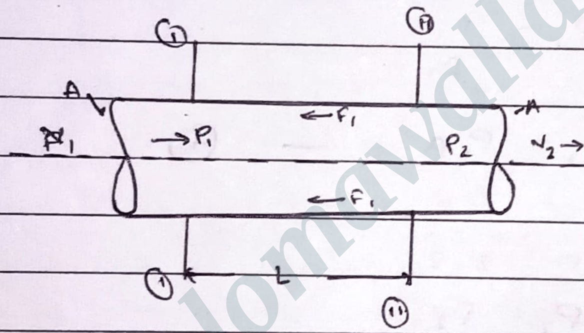
Putting $\boxed{\frac{f'}{\rho} = \frac{f}{2}}$, where f = co-efficient of friction

$$h_f = \frac{f}{2g} \times \frac{4 \times L \times V^2}{d}$$

$$\boxed{h_f = \frac{4fLV^2}{d \times 2g}} \rightarrow \text{Darcy - weisbach eq}^n$$

loss of head due to friction.

Chezy's Formula



From Darcy formula,

$$h_f = \frac{4fLv^2}{2gd}$$

h_f = loss of head due to friction
A = area of cross section of pipe $\left(\frac{\pi}{4} \times d^2\right)$
v = mean velocity of flow
P = wetted perimeter (πd)
f = co-efficient of friction.
g = gravity

$$h_f = \frac{4fLv^2}{2gd}$$

$$v^2 = \frac{h_f \times 2gd}{4 \times f \times L}$$

$$v^2 = \frac{h_f}{L} \times \frac{d}{4} \times \frac{2g}{f}$$

we know that,

hydraulic gradient;

$$i = \frac{h_f}{L}$$

& hydraulic mean depth $= m = \frac{d}{4}$

$$v^2 = i \times m \times \frac{2g}{f}$$

$$v = \sqrt{im \times \frac{2g}{f}}$$

$$v = \sqrt{i \times m} \times \sqrt{\frac{2g}{f}}$$

$$\boxed{v = c \times \sqrt{m \times i}}$$

where,

$c = \sqrt{\frac{2g}{f}}$

chezy constant, depended on value of f.

Proved