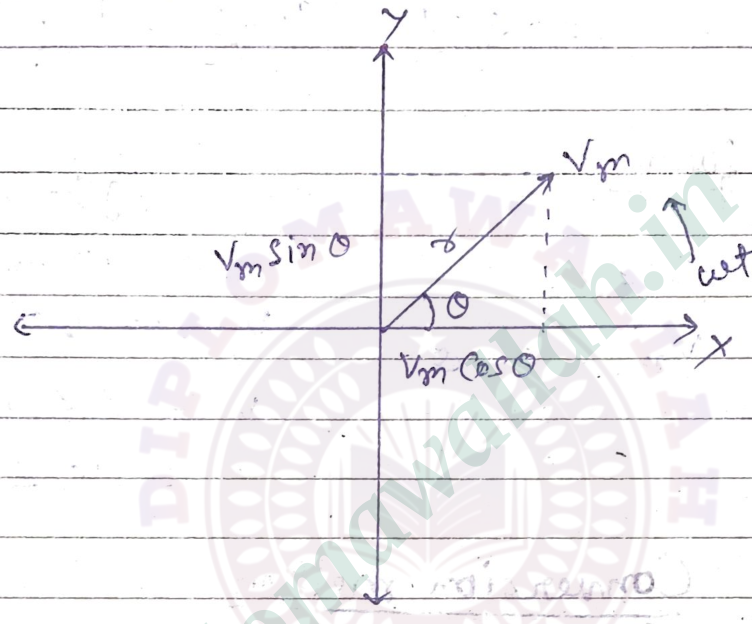
Vectorial representation of AC quantities.

Q. Represent vectors in Rectangular, Trigonometric and polar forms

$v = V_m \sin(\omega t + \theta)$ $\qquad$ $i = I_m \sin(\omega t + \theta)$



1. Rectangular form

$v = V_m \sin(\omega t + \theta)$

$v = x + iy$

or

$v = x + jy$

$x = V_m \cos\theta$

$y = V_m \sin\theta$

$V = V_m \cos\theta + i \sin\theta$

$v = V_m \cos\theta - i \sin\theta$

$v = 10 \sin(100\pi t + 50^\circ)$

eg :- ~~$v = V_m \sin(\omega t + \theta)$~~

$v = x + iy$

$x = 10 \cos 50^\circ$

$y = 10 \sin 50^\circ$

$v = 10 \cos 50^\circ + i(10 \sin 50^\circ)$

2. Trigonometric form

$$v = V_m \sin(\omega t + \theta)$$

$$V = V_m e^{j\theta} \text{ or } V_m e^{i\theta}$$

$$e^{j\theta} = \cos\theta + j\sin\theta$$

eg:- $v = 10\sin(100\pi t + 50^\circ)$

$v = V_m e^{j\theta}$

$\theta = \frac{50\times\pi}{180} = \frac{5\pi}{18}$

$v = 10 e^{j\frac{5\pi}{18}}$

3. Polar form

$$V = r\angle\theta$$

$$r = V_m \quad \theta = \theta$$

eg:-

$v = 10\sin(100\pi t + 50^\circ)$ volt

$V_m = 10,\ \omega t = 100\pi,\ \theta = 50^\circ$

$v = r\angle\theta$

$= 10\angle 50^\circ$

Conversion method

Polar to Rectangular -

Polar form - $V_m \angle\theta$

$x = V_m\cos\theta$

$y = V_m\sin\theta$

Rectangular form - $V_m\cos\theta + iV_m\sin\theta$

eg:-

Rectangular to pole :-

Rectangular - $V_m \cos\theta + i V_m \sin\theta$

$x + iy$

$x = V_m \cos\theta$, $y = V_m \sin\theta$

$r = \sqrt{x^2 + y^2}$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

Polar - $r \angle \theta$

eg:-

II Arithmetic Operations on vectors :-

1. Addition / Subtraction in Rectangular form :-

Direct addition or subtraction of corresponding real and imaginary parts.

eg :- $(3 + j4) + (1 + j2) = (3+1) + j(4+2)$
$= 4 + j6$

2. Multiplication / Division in polar forms :-

Multiplication :- $|V_1| |V_2| \angle (\theta_1 + \theta_2)$

Division :- $\left|\frac{V_1}{V_2}\right| \angle (\theta_1 - \theta_2)$

Example problems :-

1. Addition in Rectangular form :-

$V_1 = 3 + j4$

$V_2 = 1 + j2$

$V_1 + V_2 = 3 + j4 + 1 + j2$

$= 4 + j6$

2. Multiplication in polar form :-

$V_1 = 5 \angle 30^\circ$

$V_2 = 2 \angle 45^\circ$

$5 \times 2 \angle (30^\circ + 45^\circ)$

$= 10 \angle 75^\circ$

Power in AC circuit

In AC circuit there are three types of power

1. Apparent power (S)
2. Active power (P)
3. Reactive power (Q)

let V = Rms value of applied voltage
I = current flowing in AC circuit
ϕ = angle between angle V and current I

1. Apparent power (S) :-

It is the product of R.M.S. value of the applied voltage and circuit current. It is also known as wattless or idle power.

$$S = V \cdot I \quad \text{(volt-amp)}.$$

2. Active power (P) :-

It is the power which is developed in the circuit resistance. It is also known as real power.

$$P = VI\cos\phi \quad \text{(watt)}$$

$$= VI\frac{R}{Z} = \frac{V}{Z} \cdot I \cdot R$$

$$= I \cdot I \cdot R$$

$$\boxed{P = I^2 R}$$

3. Reactive power (Q) :-

It is the power which is developed ~~by~~ in the circuit reactance.

$$Q = VI\sin\phi \quad \text{(VAR)}$$

$$= V \cdot I \cdot \frac{X}{Z} = \frac{V}{Z} \cdot I \cdot X$$

$$= I \cdot I \cdot X = I^2 X$$

$$\boxed{Q = I^2 X}$$

Relation between Active power, Reactive power and aparent power.

$S = VI$

$P = VI \cos\phi$

$\varphi = VI \sin\phi$

$$P^2 + \varphi^2 = (VI \cos\phi)^2 + (VI \sin\phi)^2$$

$$P^2 + \varphi^2 = (VI)^2 (\cos^2\phi + \sin^2\phi)$$

$$P^2 + \varphi^2 = S^2 (1)$$

$$P^2 + \varphi^2 = S^2$$

$$\boxed{S = \sqrt{P^2 + \varphi^2}}$$

Power factor :-

The cosine angle of voltage and current is known as power factor. It is denoted by $\cos\phi$.

or

The ratio of active power and apparent power is known as power factor.

$$\cos\phi = \frac{KW}{KVA} = \frac{\text{Active power}}{\text{Aparent power}}$$

A resistance of 20 Ohm, inductance of 0.2 H and a capacitance of 150 microfarad are connected in series to a 230 V, 50 Hz a.c. supply. find:

Power factor 2. Active power 2. Reactive power Impedance.

$R = 20\,\Omega$, $L = 0.2\,H$, $C = 150\,\mu F$,
$V = 230\,V$, $f = 50\,Hz$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \angle \tan^{-1}\frac{(X_L - X_C)}{R}$$

$$X_L = 2\pi f L = 2\pi \times 50 \times 0.2$$

$$\boxed{X_L = 62.83\,\Omega}$$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 150 \times 10^{-6}}$$

$$\boxed{X_C = 21.22}$$

$$X_L - X_C = 62.83 - 21.22$$
$$= 41.61\,\Omega$$

$$Z = \sqrt{(20)^2 + (41.61)^2} \angle \tan^{-1}\frac{41.61}{20}$$

$$Z = 46.16 \angle 64.32^\circ\,\Omega$$

$\phi = 64.32°$

① Power factor = $\cos\phi = \cos(64.32)$

$$\boxed{\cos\phi = 0.433}$$

$\because X_L > X_C$ so the power factor will be lagging.

2. Active power = $VI\cos\phi$

$$= 230 \times \left(\frac{V}{|Z|}\right) \times 0.433$$

$$= 230 \times \left(\frac{230}{46.16}\right) \times 0.433$$

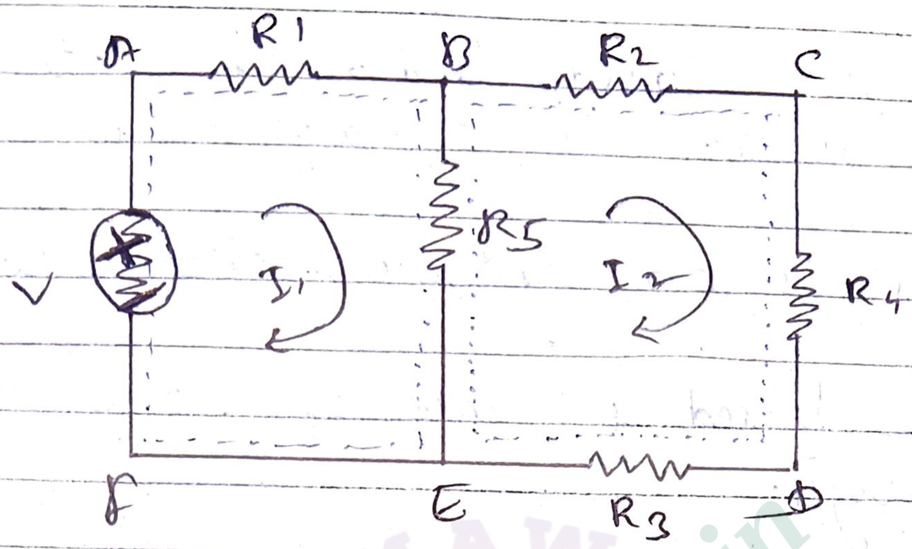
$$\boxed{P = 496.22\ W}$$

3. Reactive power = $VI\sin\phi$

$$= 230 \times \frac{230}{46.16} \times \sin(64.32)$$

$$\boxed{Q = 1032.82\ VAR}$$

Mesh and loop :-



Mesh :- It is a close path which does not have any further close path.

eg :- ABEFA, BCDEB

Loop :- It also a close path which may or may not have any further close path.

eg :- ABCDEF, ABEF, BCDE.

Kirchhoff's law

- KVL
- KCL

① **Kirchhoff's voltage law (KVL) :-**

→ In a closed loop, the total sum of voltage is always zero.

→ This is an interconnection law.

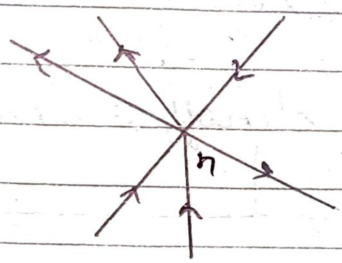
→ KVL based on law of conservation of energy.

② **Kirchhoff's current law (KCL) :-**

→ Based on conservation of charge.

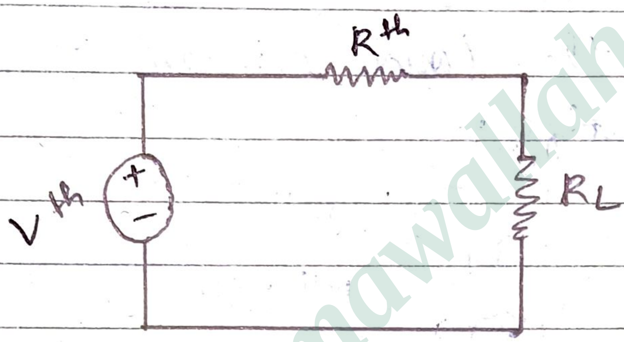
→ Total incoming current = Total outgoing current

→ At a node, the sum of total current is always zero.



Thevenin's theorem :-

According to the thevenin's theorem. Any linear bilateral network irrespective of its complexities can be reduced into a thevenin's equivalent circuit having the thevenin's open circuit voltage 'V_{th}' in series with the thevenin's equivalent resistance 'R_{th}' along with load resistance or R_L.



$$\boxed{I_L = \frac{V_{th}}{R_{th} + R_L}}$$

Steps for solving :-

→ Identify the load resistor R_L

→ Remove the load resistor and calculate the open circuit resistor potential across the two operands. this will be the thevenin's voltage V_{th}.

→ Again remove the load resistor and replace all the active sources by their internal resistance.

→ Calculate the equivalent resistance across the operands: this will be the thevenin's equivalent resistance R^{th}.

→ Draw the thevenin's equivalent for given network.

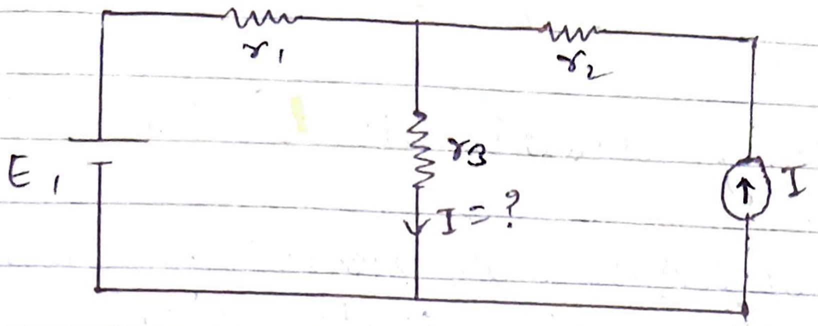
→ Calculate the load current I_1 by using the identify:

$$\boxed{I_1 = \frac{V^{th}}{R^{th} + R_L}}$$

Superposition theorem :-

If there are two or more than two sources (EMF or current) acting simultaneously in a linear network, the current flowing through any branch is the algebraic sum of all the current which should flow in that branch. if (EMF or current) considered saparetly.

limitation → At least two sources are present.



Case - I E_1 ✓ I ✓ $r_3 = I$

Case - II E_1 ✓ I ✗ (O.C.) $r_3 = I'$

Case - III E_1 ✗ (S.C) I $r_3 = I''$

$$\therefore \boxed{I = I' + I''}$$

Power system

1. Generation
2. Transmission and distribution
3. Switchgear & protection

Important points :-

☆ Generation :- Generally in multiple of 11 kV.

☆ Primary Transmission :- 765 kV, 400 kV, 220 kV

☆ Secondary Transmission :- 132 kV, 66 kV, 33 kV

* Primary distribution :- 11 kV, 6.6 kV, 3.3 kV

* Secondary distribution :- 415 V / 230 V

- Transformer connected to generator is step up (Delta star)

- In secondary distribution transformer is step down

- In transmission line circuit and primary distribution transformer is step down (star delta).

- Size of feeder is depends on current carrying capability.

- Size of Distributor is depends on voltage drop.

- Conductor connecting Distributor to energy meter is called service mains.

- Conductor connecting substation to Distribution transformer is called feeder.

- Conductor connecting and distribution Transformer is called Distributor.

Advantage of high voltage Transmission

→ Maximum power transfer capability will increase.

→ Transmission line losses will reduce. $P_L \propto \frac{1}{V^2}$

→ Area and volume of conductor is minimize.

→ Cost will reduce.

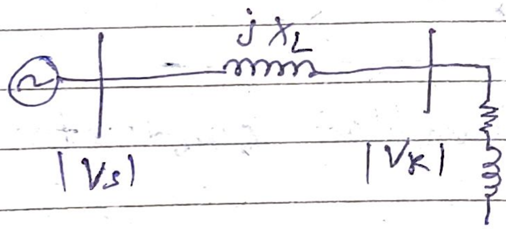
NOTE :- At very high voltage level the corona loss will becomes very high.

We can also increase maximum power transfer capability by reducing "X_L"

$$(P_R)_{max} = \frac{V_s^2}{X_L}$$

$$\uparrow (P_R)_{max} \propto \frac{1}{X_L \downarrow}$$

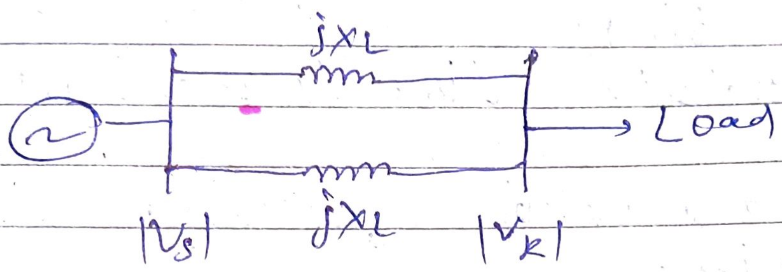
Methods to reduce "X_L"



$$\boxed{(P_R)_{max} = \frac{V_s \cdot V_R}{X_L}}$$

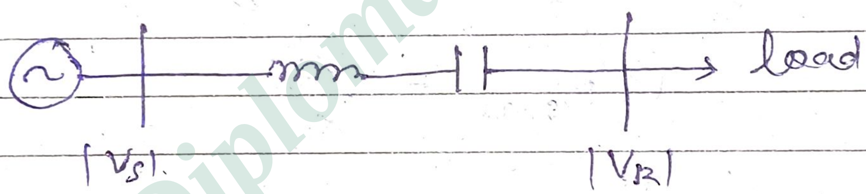
1. By using parallel line

$$x_L \| x_L = \frac{x_L}{2}$$



$$\boxed{(P_R)_{max_2} = \frac{V_S \cdot V_R}{X_L/2} = \frac{2 V_S V_R}{X_L}} = 2\,(P_{max})_1$$

2. By using series capacitor



$$(P_R)_{max} = \frac{V_S \cdot V_R}{X - \frac{X}{2}} = \frac{V_S \cdot V_R}{X/2}$$

$$\boxed{(P_R)_{max} = \frac{2 V_S \cdot V_R}{X}}$$

3. By using Bundled conductor

4. By using double circuit line.

\# For stability purpose load angle should be 30 degree to 45 degree.

Transmission lines :-

Transmission lines are used for the transmission of electrical energy or signals, typically over long distances.

Transmission lines connect power plants to substations and eventually to end - users.

Types of Transmission lines (Based on length)

$f = 50\,Hz$

① Short Transmission line →

$l \rightarrow$ length
f = frequency

length < 80 km

$lf < 4000$

② Medium Transmission line →

$80\,km \leq l \leq 200\,km$

$4000 \leq lf \leq 10000$

③ Long Transmission line →

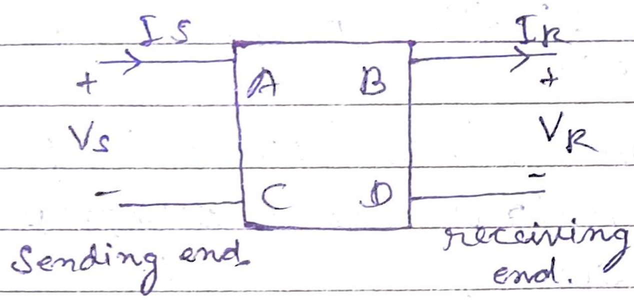
$l > 200$ km

$lf > 10000$

ABCD Parameter

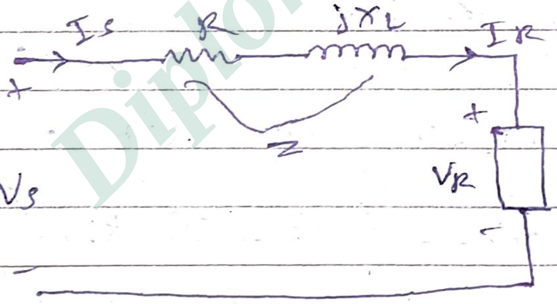
$f = 50\,Hz$, but 3% variation allowed ($51.5\,Hz$ to $48.5\,Hz$)

→ It is also called Transmission line parameter.



$$\boxed{\begin{aligned} V_S &= AV_R + BI_R \\ I_S &= CV_R + DI_R \end{aligned}}$$

- Find ABCD parameter of short transmission line −



$V_S = V_R + ZI_R$

$V_S = AV_R + BI_R$

$\boxed{A = 1 \text{ \& } B = Z}$

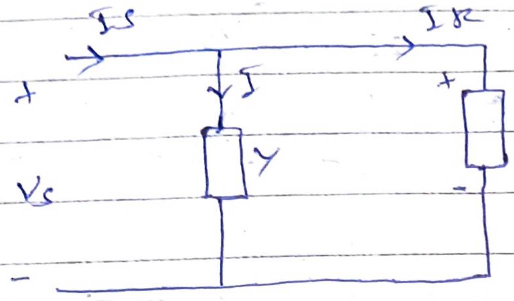
$I_S = I_R$

$I_S = CV_R + DI_R$

$\boxed{C = 0}$ $\boxed{D = 1}$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}$$

• ABCD parameter of admitance circuit



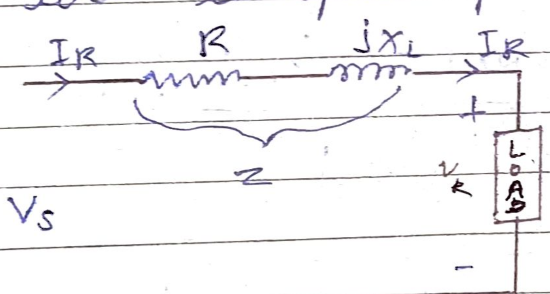
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix}$$

Short Transmission line :-

→ If length less than 80 km ($l < 80$ km)
or $lf < 4000$.

→ In this Transmission line only R & L is consider.

→ In this Transmission line R & L are lumped parameter.



where
R → Total per phase resistance of Transmission line.
L → Total per phase inductance of line.

ABCD parameter -

$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

Losses -

$$\boxed{\begin{aligned} \text{Loss } (P_L) &= I_R^2 \cdot R \quad\quad 1-\phi \\ (P_L) &= 3 I_R^2 R \quad\quad 3-\phi \end{aligned}}$$

Efficiency :-

$$\eta = \frac{\text{output power}}{\text{output power} + \text{loss}}$$

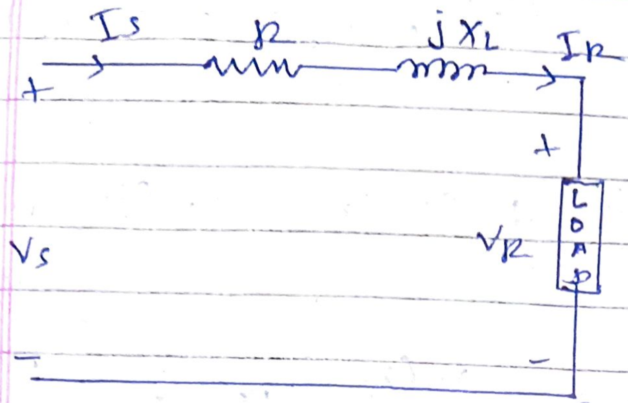
$$\boxed{\text{output power } (P_R) = \sqrt{3}\, V_R \cdot I_R \cos\phi_R}$$

Voltage Regulation :-

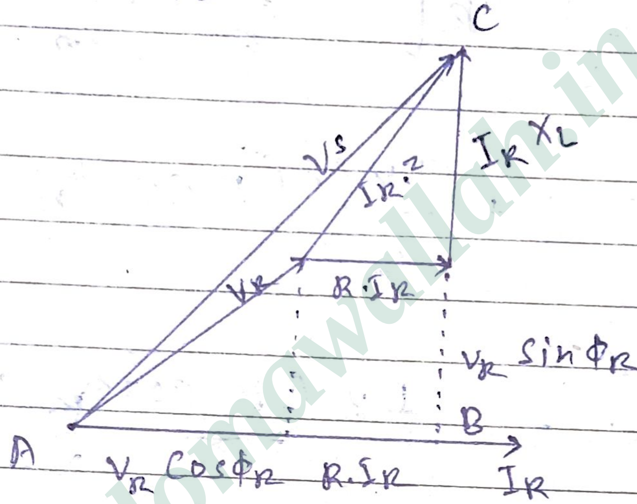
$$V.Reg = \frac{|V_{NL}| - |V_{FL}|}{|V_{FL}|} \times 100$$

$$\boxed{V.Reg = \frac{|V_s| - |V_R|}{|V_R|} \times 100}$$

Lagging load.



Phasor diagram −



On A ΔABC

$$AC^2 = AB^2 + BC^2$$

$$V_s^2 = (V_R \cos\phi_R + I_R \cdot R)^2 + (V_R \sin\phi_R + X_L \cdot I_R)^2$$

$$= V_R^2(\cos^2\phi_R + \sin^2\phi_R) + 2V_R I_R(R\cos\phi_R + X_L \sin\phi_R) + I_R^2(R^2 + X_L^2)$$

$$= V_R^2 + 2V_R I_R(R\cos\phi_R + X_L \sin\phi_R) + \underbrace{I_R^2(R^2 + X_L^2)}_{\text{Neglect}}$$

$$V_s^2 = V_R^2 + 2V_R I_R(R\cos\phi_R + X_L \sin\phi_R)$$

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$$V_S = \left[V_R^2 + 2V_R I_R (R\cos\phi_R + X_L \sin\phi_R)\right]^{1/2}$$

$$V_S = V_R\left[1 + \frac{2I_R}{V_R}(R\cos\phi_R + X_L\sin\phi_R)\right]^{1/2}$$

$$(1+x)^n = \left(1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots\ldots\right)$$

$$V_S = V_R\left[1 + \frac{1}{2}\cdot\frac{2I_R}{V_R}(R\cos\phi_R + X_2\sin\phi_R)\right]$$

$$\frac{V_S}{V_R} = 1 + \frac{I_R}{V_R}(R\cos\phi_R + X_L\sin\phi_R)$$

$$\frac{V_S}{V_R} - 1 = \frac{I_R(R\cos\phi_R + X_L\sin\phi_R)}{V_R}$$

$$\frac{V_S - V_R}{V_R} = \frac{I_R(R\cos\phi_R + X_L\sin\phi_R)}{V_R}$$

$$\boxed{V.Reg. = \frac{I_R(R\cos\phi_R + X_L\sin\phi_R)}{V_R}}$$ Lagging load.

<u>In General (for all Transmission line)</u>

$$V_S = AV_R + BI_R$$

at no load $I_R = 0$

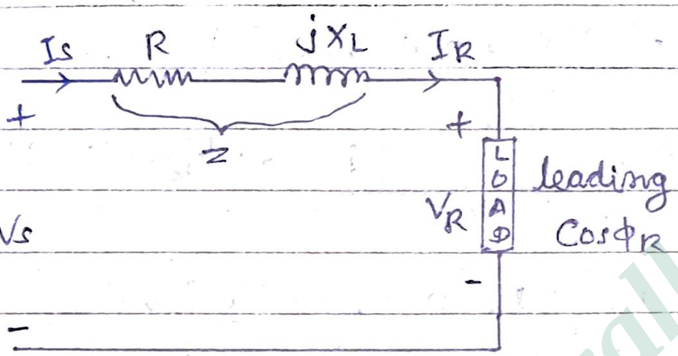
$$V_S = AV_R$$

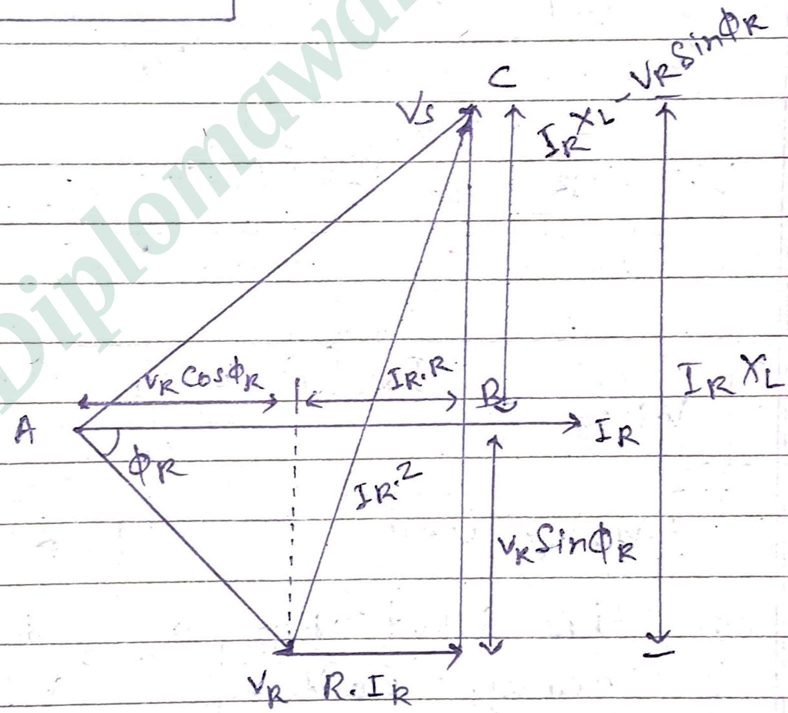
v.v.i

$$\boxed{(V_R)_{N.L} = \frac{V_S}{A}}$$

$$\boxed{V.Reg = \frac{\left|\frac{V_S}{A}\right| - |V_R|}{|V_R|} \times 100}$$

leading load –





In a Δ ABC

$$AC^2 = AB^2 + BC^2$$

$$V_S^2 = (V_R \cos\phi_R + I_R \cdot R)^2 + (I_R X_L - V_R \sin\phi_R)^2$$

$$V_S^2 = V_R^2(\cos^2\phi_R + \sin^2\phi_R) + 2V_R \cdot I_R(R\cos\phi_R - X_L\sin\phi) + \underbrace{I_R^2(R^2 + X_L^2)}_{\text{Neglect}}$$

$$V_S^2 = V_R^2 + 2V_R I_R (R\cos\phi_R - X_L\sin\phi_R)$$

$$V_S = \left[V_R^2 + 2V_R I_R (R\cos\phi_R - X_L\sin\phi_R)\right]^{1/2}$$

$$\because (1+x)^n = 1 + nx + \ldots\ldots$$

$$V_S = V_R\left[1 + \frac{1}{2}\frac{2I_R}{V_R}(R\cos\phi_R - X_L\sin\phi_R)\right]$$

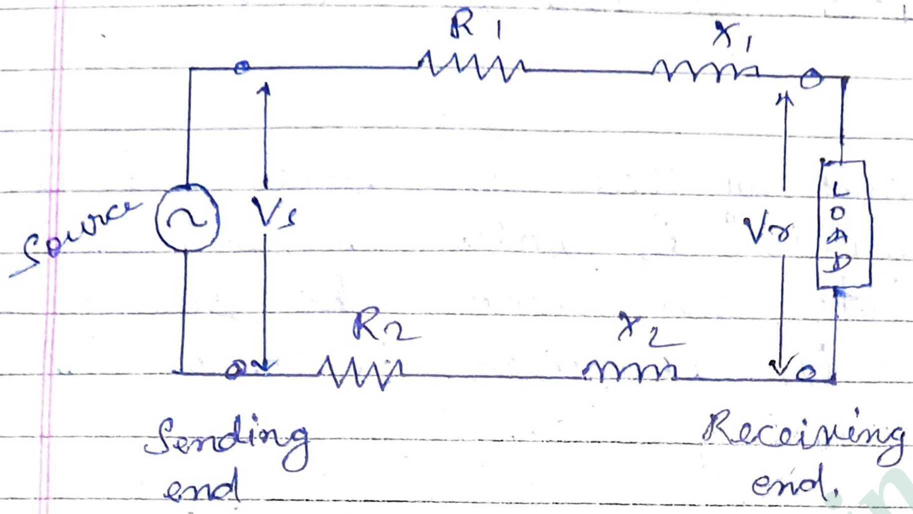
$$\frac{V_S}{V_R} - 1 = \frac{I_R(R\cos\phi_R - X_L\sin\phi_R)}{V_R}$$

$$\frac{V_S - V_R}{V_R} = \frac{I_R(R\cos\phi_R - X_L\sin\phi_R)}{V_R}$$

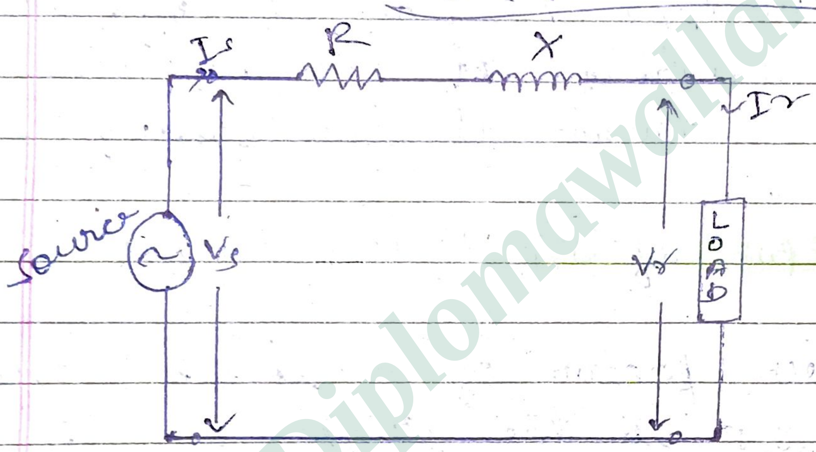
$$\boxed{V.Reg = \frac{I_R(R\cos\phi_R - X_L\sin\phi_R)}{V_R}}$$

Lagging load	Leading load
→ In this load V.Reg is always positive.	→ If $R\cos\phi_R = X_L\sin\phi_R$, V.Reg $= 0$
→ & also the maximum voltage regulation will occurs.	→ If $R\cos\phi_R > X_L\sin\phi_R$, V.Reg $= +ve$
	→ If $R\cos\phi_R < X_L\sin\phi_R$, V.Reg $= -ve$.

Equivalent Circuit of transmission line :-



Single phase line



Equivalent circuit model of a short line.

where,

R = loop resistance of the line = resistance of both outgoing and return conductors

= 2 X resistance of one conductor = $2R_1$

X = loop reactance of the lines = reactance of both lead and return conductors.

= 2 × inductive reactance to one conductor to neutral = $2X_1$

The end of the line where the load is connected is called receiving end. The end where the source of supply is connected is known as the sending end.

let V_r = Voltage at the receiving end
V_s = voltage at the sending end
I_r = current at the receiving end.
I_s = current at the sending end.

Power factor of Short transmission line :-

⇒ The power factor of a short transmission line refers to the cosine of the angle between the sending-end voltage (V_s) and the line current (I). It is defined as :-

Power factor (PF) = $\cos\phi$

where ϕ is the phase angle between V_s and I.

Equation for receiving end voltages:-

$$V_S = V_R + Z \cdot I$$

$$V_S = V_R + (R + jX) \cdot I$$

$$V_R = V_S - (R + jX) \cdot I$$

Here,

V_R = Receiving end voltage
V_S = Sending end voltage

- $Z = R + jX$ = Series impedance of the line
  I = Line current.

$$|V_R| = \sqrt{Re(V_R)^2 + Im(V_R)^2}$$

Resistance (R) :-

- Represents the opposition to current flow in the conductor.

- Depends on conductor material, cross-sectional area, and length.

Inductance :-

- Represents the magnetic field energy storage.
- Depends on conductor spacing, geometry and current.

Capacitance :-

- Represents the ability of the line to store electrical energy in its electric field.

- Exists between the conductors and ground.

Conductance :-

- Represents leakage current between conductors.
- Usually negligible for overhead lines but significant for underground cables.

Ferrantic effect :-

During no load condition or lightly load condition. In case of medium and long transmission line the receiving end voltage becomes more than the sending end voltage, this is called ferrantic effect.

$$\boxed{V_R > V_s}$$

→ During ferrantic effect condition source will operates at leading power factor.

How to reduce the ferranti effect :-

⇒ 1. Shunt Reactors :-

- Install shunt reactors at the receiving end or along the transmission line.
- These absorb reactive power and reduce the voltage rise.

2. Series Capacitors :-

- Use series capacitors to compensate for the line inductance, helping control voltage levels.

3. Voltage Regulators :-

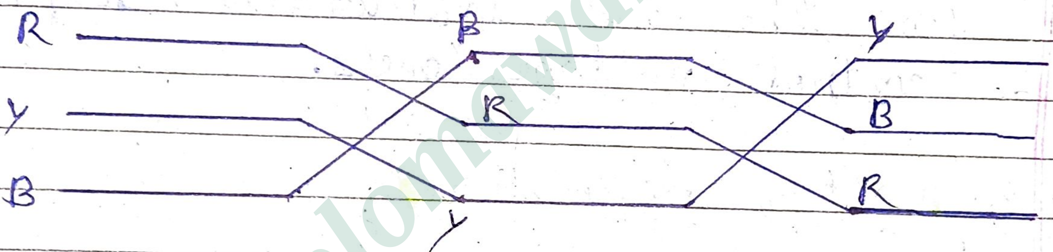
Employ voltage regulators or tap-changing transformers to maintain voltage levels within permissible limits.

4. Shortening Transmission line :-

Divide long transmission lines into shorter segments with intermediate substations.

Transposition of conductor :-

Transposition of conductor is the periadic swapping of position of conductar of a transmission line to maintain voltage drop in every wire because of mutual inductance swapping is mandatory and magnetic field exists in the conducto which cause of interference in the line this can be reduced by continuously swapping of conductor.



Advantages :-

① Balancing capacitance :-

Transposition balances the overall capacitance for the entire line by periodically rotating the positions of the conductors. This is necessary because capacitance between conductors and ground is not typically symmetrical across phases.

(2) Reducing voltage drops :-

Transposition reduce voltage drops caused by the inductance of an unsymmetrical line.

(3) Preventing interference :-

Transposition prevents interferences with neighboring telephone lines.

(4) Reducing power loss :-

Transposition reduces system power loss in high voltage lines.

(5) Reducing electrostatic unbalanced voltages :-

Transposition reduces the effect of electrostatic unbalanced voltages.

Skin effect :-

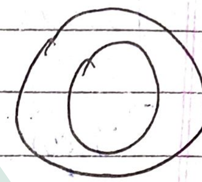
In case of DC supply current will uniformly flow througout the cross-section, but in case of AC supply current want to flow on the surface of conductor, due to this the resistance of AC will becomes more than DC.

OR.

Non-uniform distribution of current in conductor is called skin effect.

$$\boxed{\text{Skin effect} \propto \sqrt{\pi \sigma \mu f} \cdot d^2}$$

where,



μ = Permeability
d = diameter
f = frequency
σ = Conductivity

$\boxed{\rho = \frac{1}{\sigma}}$ ← resistivity.

Proximity effect :-

→ ~~we~~ The current flowing through the conductor, the flux will produced ~~&~~ this flux links with adjacent conductor due to this a non-uniform distribution of current will occurs in adjacent conductor this is called proximity effect.