Unit :- 03

Filters

Day :- Thursday   Date :- 20/12/24

Filter...

A filter is a circuit that is design to pass signals with desired frequencies and reject or attenute others.

Multiple Frequency → | Filter | → Desired frequency.

» Classification of Filter...

Filter
- Analog or Digital Filter
- Active or Passive filter
- Audio (AF) or Radio (RF) Filter

Passive Filter...

Passive filter are those filter which has only passive components such as resistor, inductor, capacitor etc.

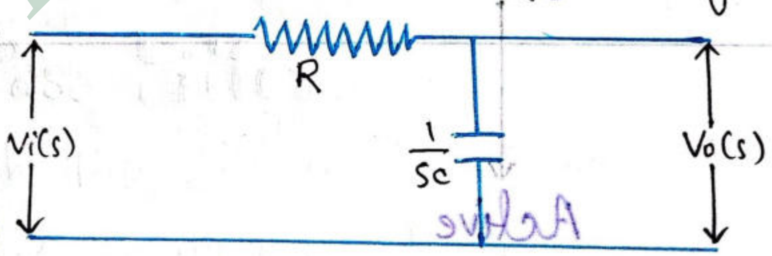
→ They are classified into four types :-

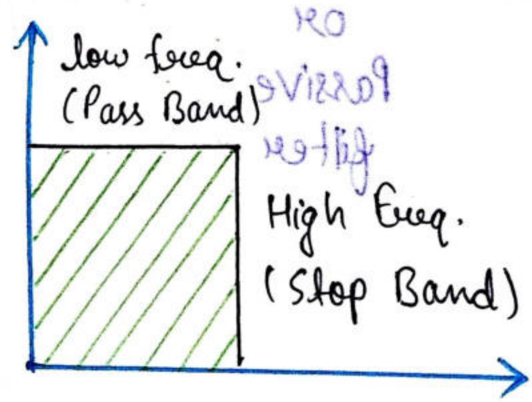
i) Low Pass Filter
ii) High Pass Filter
iii) Band Pass Filter
iv) Band Stop Filter

(i) Low Pass Filter...

In low pass filter it allows only low frequencies components that means it reject all other high frequency component.

• The s-domain circuit diagram of LPF :-





→ The circuit consist of two passive component resistor and capacitor, connected in series.

→ Here Vi(s) and Vo(s) are the laplace transform of input voltage Vi(t) and output voltage Vo(t) respectively.

- Transfer Function :- |H(s)|

$$|H(s)| = \frac{V_o(s)}{V_i(s)}$$

$$= \frac{\frac{1}{sC}}{R + \frac{1}{sC}}$$

$$= \frac{\frac{1}{\cancel{sC}}}{\frac{sCR+1}{\cancel{sC}}}$$

$$\boxed{|H(s)| = \frac{1}{sCR+1}}$$

$\boxed{* \text{ Note } j^2 = -1}$

→ Putting $s = j\omega$

$$|H(s)| = \frac{1}{j\omega CR + 1}$$

$$|H(s)| = \frac{1}{\sqrt{(\omega CR)^2 + 1}}$$

- at $\omega = 0$, magnitude $= 1$

$$\frac{1}{\sqrt{(0 \times CR)^2 + 1}} = \frac{1}{0+1} = \frac{1}{1} = 1.$$

- At $\omega = \frac{1}{RC}$, magnitude $= \frac{1}{\sqrt{2}} = 0.707$

$$\frac{1}{\sqrt{\left(\frac{1}{RC}\times RC\right)^2+1}} = \frac{1}{\sqrt{1^2+1^2}} = \frac{1}{\sqrt{2}}$$

- At $\omega = \infty$, magnitude $= 0$

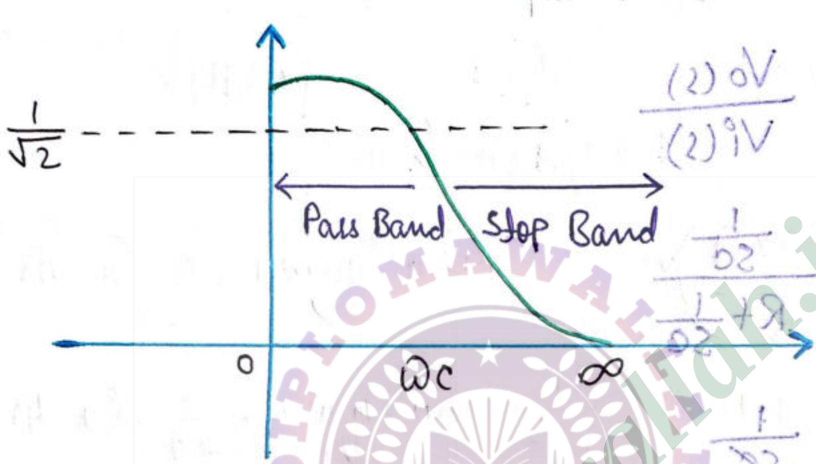


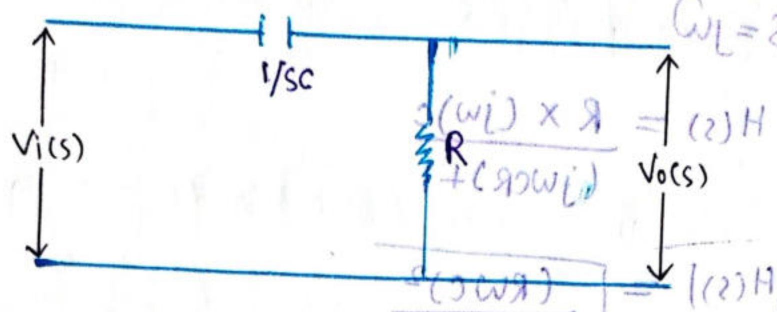
Fig:- Frequency response diagram

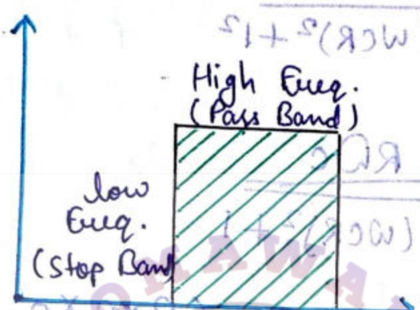
- Low Pass filter magnitude vary from 1 to 0 as ω (omega) varies from 0 to ∞

ii) High Pass Filter:-

In high pass filter it allows only ~~low~~ high frequency components that means it reject all other low frequency components.

- The s-domain circuit diagram of HPF.





→ This circuit consist of two passive component resistor and capacitor connected in series.

→ Here Vi(s) and Vo(s) are the laplace transform of input voltage vi(t) and output voltage Vo(t) respectively.

- Transfer Function :-

$$|H(s)| = \frac{V_o(s)}{V_i(s)}$$

$$H(s) = \frac{R}{R+\frac{1}{SC}}$$

$$|H(s)| = \frac{R}{\frac{RSC+1}{SC}} = \frac{R}{1} \times \frac{SC}{RSC+1}$$

$$|H(s)| = \frac{RSC}{SCR+1}$$

Putting $s = j\omega$

$$H(s) = \frac{R \times (j\omega)C}{(j\omega CR)+1}$$

$$|H(s)| = \sqrt{\frac{(R\omega C)^2}{(\omega CR)^2+1^2}}$$

$$|H(s)| = \frac{R\omega C}{\sqrt{(\omega CR)^2+1}}$$

- At $\omega = 0$, magnitude $= 0 \ldots \therefore \frac{R \times 0 \times C}{\sqrt{(0 \times CR)^2+1}} = \frac{0}{0+1} = \frac{0}{1} = 0$

- At $\omega = \frac{1}{RC}$, magnitude $= \frac{1}{\sqrt{2}} = 0.707$

$$\frac{\frac{1}{RC} \times RC}{\sqrt{\left(\frac{1}{RC} \times RC\right)^2+1}} = \frac{1}{\sqrt{1^2+1^2}} = \frac{1}{\sqrt{2}}$$

- At $\omega = \infty$, magnitude $= 1 \ldots \therefore \frac{R\infty C}{\sqrt{(\infty CR)^2+1}}$

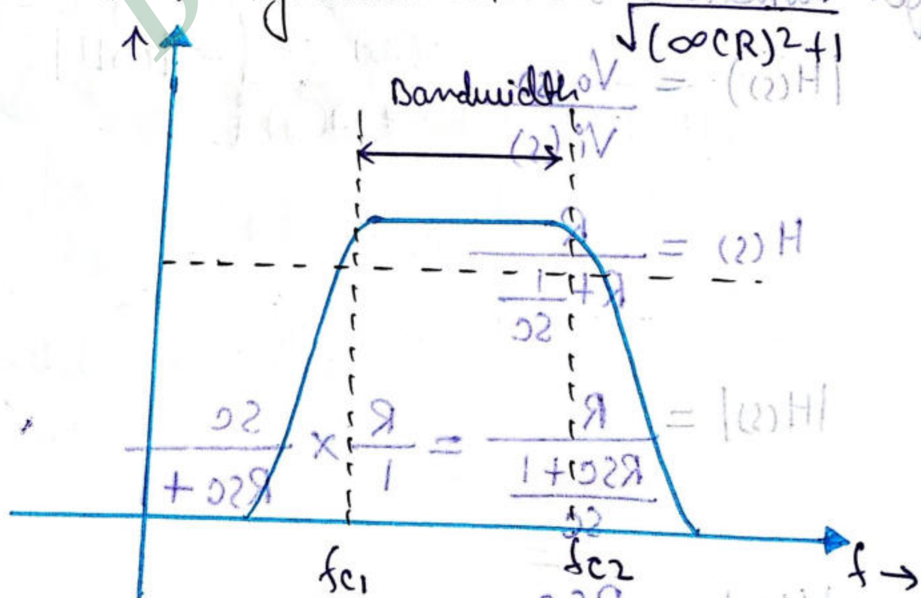


Fig:- Frequency Response diagram

- High Pass filter magnitude vary from 0 to 1 as ω (omega)

Date :- 08/01/25 Day :- Wednesday.

(iii) Band Pass Filter (BPF) :-

In Band pass filter, it allows only one band of frequency. This frequency band lies between low frequency range and high frequency range. It means this filter reject both high frequency and low frequency.

- The S-domain circuit diagram :-

→ This circuit consist of three passive element Inductor, capacitor and resistor connected in series.

→ $V_i(s)$ and $V_o(s)$ are laplace transformed of input voltage $V_i(t)$ and output voltage $V_o(t)$ respectively.

• Transfer Function :- $|H(s)| = \frac{V_o(s)}{V_i(s)}$

$$= \frac{R}{R + sL + \frac{1}{sC}}$$

$$= \frac{RSC}{SCR + S^2LC + 1}$$

• Putting $S = J\omega$

$$= \frac{RSC}{SCR + S^2LC + 1}$$

$$= \frac{R(J\omega)C}{(J\omega)CR + (J\omega)^2LC + 1}$$

$$= \frac{J\omega RC}{J\omega RC + J^2\omega^2LC + 1}$$

$$= \frac{J\omega RC}{J\omega RC - \omega^2LC + 1}$$

→ Magnitude :-

$$|H(s)| = \sqrt{\frac{(\omega RC)^2}{(\omega RC)^2 + (-\omega^2LC + 1)^2}}$$

$$= \frac{\omega RC}{\sqrt{(\omega RC)^2 + (-\omega^2LC + 1)^2}}$$

• At $\omega = 0$, magnitude $= 0$

• At $\omega = \frac{1}{\sqrt{LC}}$, magnitude $= 1$.

$$= \frac{\frac{1}{\sqrt{LC}} \times RC}{(\frac{1}{\sqrt{LC}} \times RC)^2 \; (-(\frac{1}{\sqrt{LC}})^2 \times LC + 1)^2}$$

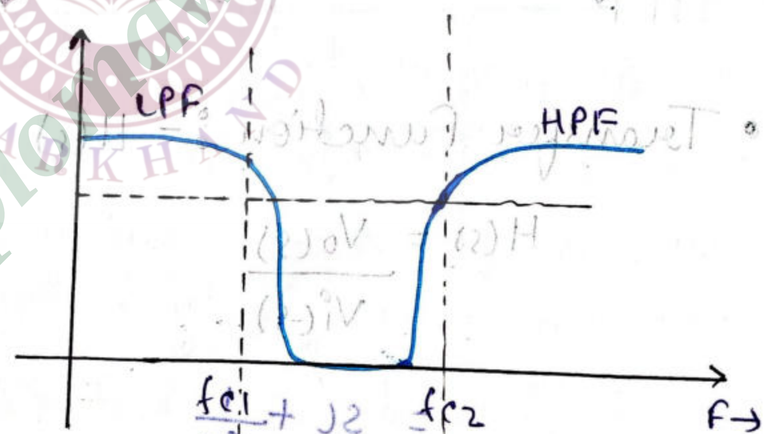
$$= \frac{\frac{1}{\sqrt{LC}} RC}{\sqrt{(\frac{1}{\sqrt{LC}} RC)^2 + (\frac{-1}{\sqrt{LC}})^{\cancel{2}} \cdot LC + 1}}$$

$$= \frac{\frac{RC}{\sqrt{LC}}}{\sqrt{(\frac{RC}{\sqrt{LC}})^2 + (\frac{-1}{\cancel{LC}} \times \cancel{LC} + 1)^2}}$$

$$= \frac{\frac{RC}{\sqrt{LC}}}{\sqrt{(\frac{RC}{LC})^{\cancel{2}}}}$$

$$= \frac{\frac{R\cancel{C}}{\cancel{\sqrt{LC}}}}{\frac{R\cancel{C}}{\cancel{\sqrt{LC}}}} = 1$$

$$= \boxed{1}$$



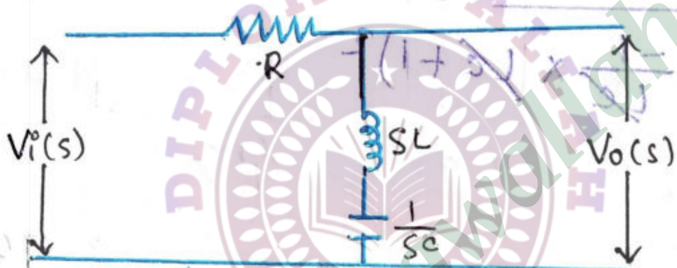
- At $\omega = \infty$, magnitude $= 1$.

(iv) Band Stop Filter... (BSF)

(Band elimination filter, Band rejection element).

In Band stop filter, it rejects only one band of frequency. In this frequency band lies in b/w low frequency range. It means it allows both low frequency and high frequency components.

- The S-domain circuit :-



- Transfer Function :- H(s)

$$H(s) = \frac{V_o(s)}{V_i(s)}$$

$$= \frac{SL + \frac{1}{SC}}{R + SL + \frac{1}{SC}}$$

$$= \frac{\frac{S^2LC + 1}{\cancel{SC}}}{\frac{SCR + S^2LC + 1}{\cancel{SC}}}$$

$$= \frac{S^2LC + 1}{SCR + S^2LC + 1}$$

• Putting $s = j\omega$

$$= \frac{(j\omega)^2 \times LC + 1}{(j\omega)RC + (j\omega)^2 \times LC + 1}$$

$$= \frac{j^2\omega^2 LC + 1}{j\omega RC + j^2\omega^2 LC + 1}$$

$$= \frac{-\omega^2 LC + 1}{j\omega RC - \omega^2 LC + 1}$$

→ Magnitude = $|H(s)|$

$$= \sqrt{\frac{(1-\omega^2 LC)^2}{(\omega RC)^2 + (1-\omega^2 LC)^2}}$$

$$= \frac{1-\omega^2 LC}{\sqrt{(\omega RC)^2 + (1-\omega^2 LC)^2}}$$

• At $\omega = 0$, magnitude = 1

• At $\omega = \frac{1}{\sqrt{LC}}$, magnitude = 0

• At $\omega = \infty$, magnitude = 1

» Characteristics of an ideal filter (LPF, HPF, BPF and BRF)

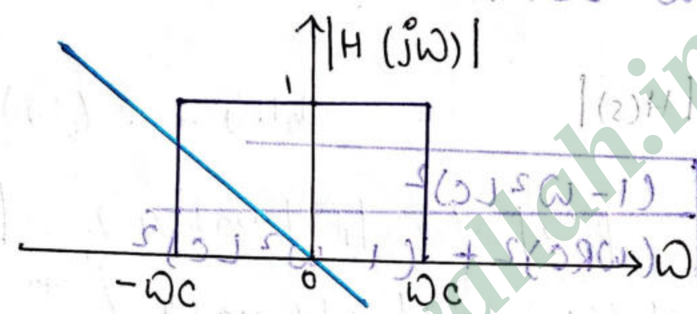
→ LPF :-

It is filter which transmits all the signal of frequencies less than a certain frequency ω_c

radians per second without any distortion above ω_c radians per second where ω_c is called cut-off freq.

$$[\theta(\omega) = -\omega t_d]$$

$$|H(\omega)| = \begin{cases} 1 & \text{for } |\omega| < \omega_c \\ 2 & \text{for } |\omega| > \omega_c \end{cases}$$

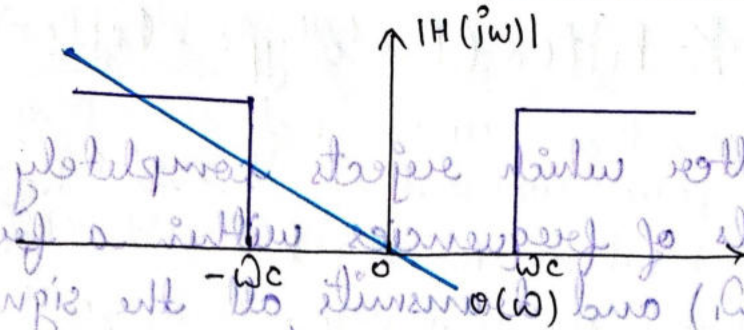


→ **HPF** :-

It is a filter which transmit all the signals of frequencies above a certain frequency ω_c radians per second without any distortion and blocks completely all the signals below freq. ω_c. Here frequency ω_c is called the cut off frequency.

$$\theta(\omega) = -\omega t_d$$

$$|H(j\omega)| = \begin{cases} 0 & \text{for } |\omega| < \omega_c \\ 1 & \text{for } |\omega| > \omega_c \end{cases}$$



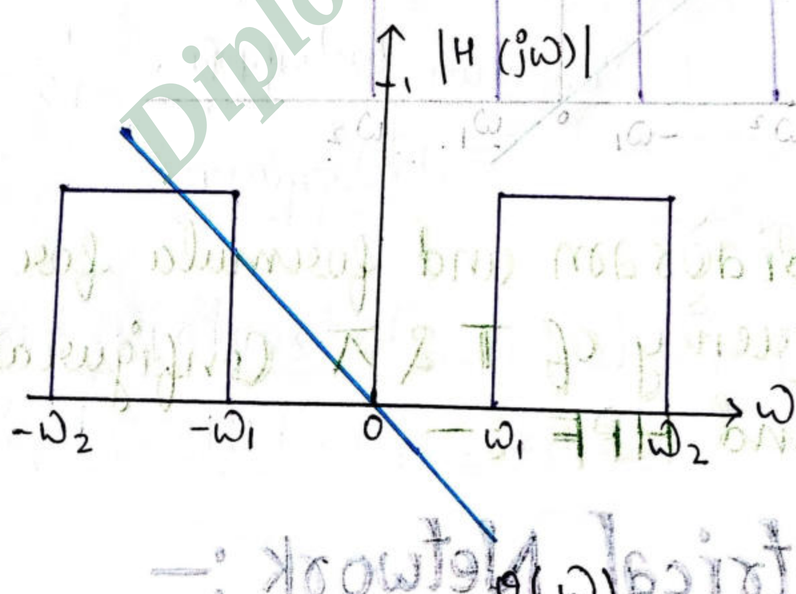
→ BPF :-

It is a filter which transmits all the signal of frequencies within a certain frequency band ($\omega_2 - \omega_1$) without any distortion and completely blocks all the signals of frequency outside this band.

$$\omega_2 - \omega_1 = \text{Bandwidth of BPF}$$

$$\theta(\omega) = -\omega t_d$$

$$|H(j\omega)| = \begin{cases} 1 & \text{for } |\omega_1| < \omega < |\omega_2| \\ 0 & \text{for } \omega < |\omega_1| \ \& \ \omega > |\omega_2| \end{cases}$$



→ **BRF :-**

It is a filter which rejects completely all the signals of frequencies within a frequency band $(\omega_2 - \omega_1)$ and transmits all the signal of frequency outside the band.

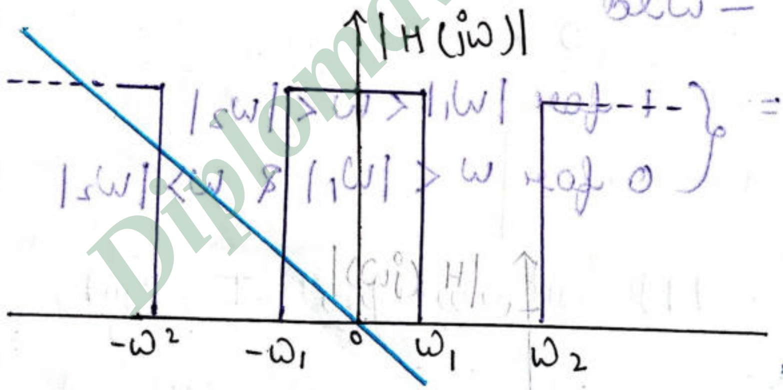
$(\omega_2 - \omega_1) =$ rejection band

or BSF, BEF.

→ Phase

$\theta(\omega) = -\omega t_d$

$$|H(j\omega)| = \begin{cases} 0 \text{ for } |\omega_1| < \omega < |\omega_2| \\ 1 \text{ for } \omega < |\omega_1| \ \& \ \omega > |\omega_2| \end{cases}$$



Circuit diagram and formula for cut-off frequency of T & π Configuration of LPF and HPF :-

≫ Symmetrical Network :-

A network is symmetrical, if its input impedence is equal to its output impedence there are the network where input and output are dual of each other.

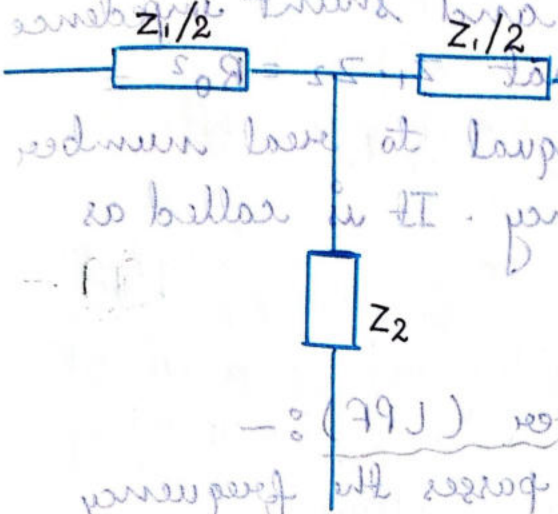


Fig:- T- Section

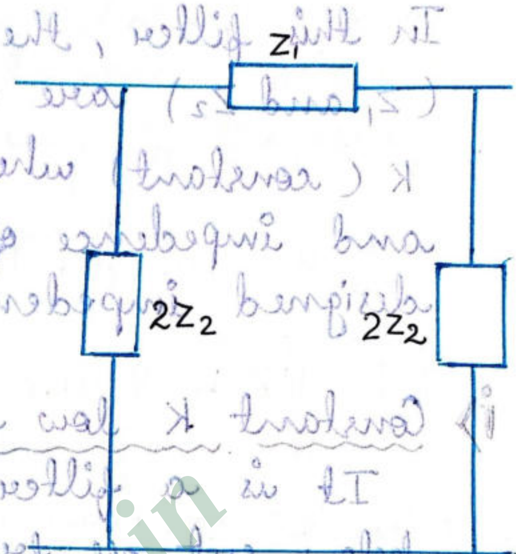


Fig:- π-Section

There are two important properties of symmetrical network, namely characteristics impedence (Z_0) and propagation constant (γ).

→ **For T-Network**

$$Z_{0T} = \sqrt{\frac{Z_1^2}{4} + Z_1 \cdot Z_2} = \text{characteristics Impedence.}$$

$$e^{\gamma} = 1 + \frac{Z_1}{2Z_2} + \frac{Z_{0T}}{Z_2} = \text{Propagation Constant.}$$

→ **For π-Network:-**

$$Z_{0\pi} = \frac{Z_1 \cdot Z_2}{\sqrt{\frac{Z_1^2}{4} + Z_1 \cdot Z_2}} = \frac{Z_1 \cdot Z_2}{Z_{0T}} = \text{Characteristics Impedence.}$$

$$e^{\gamma} = 1 + \frac{Z_1}{2Z_2} + \frac{Z_1}{Z_{0\pi}} = \text{Propagation Constant.}$$

▷ Constant K-Filter (Prototype-Filter):-

In this filter, the series and shunt impedence (z_1 and z_2) are such that $Z_1 \cdot Z_2 = R_0^2 =$ K (constant) where R_0 equal to real number and impedence of frequency. It is called as designed impedence.

i) Constant K low pass filter (LPF):-

It is a filter which passes the frequency below cut-off frequency and rejects / attenuates all other frequency above cut-off frequency.

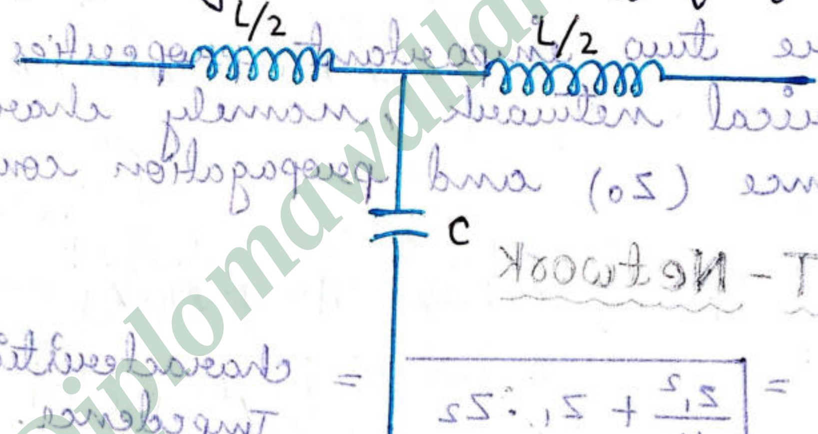


Fig:- T- Configuration LPF

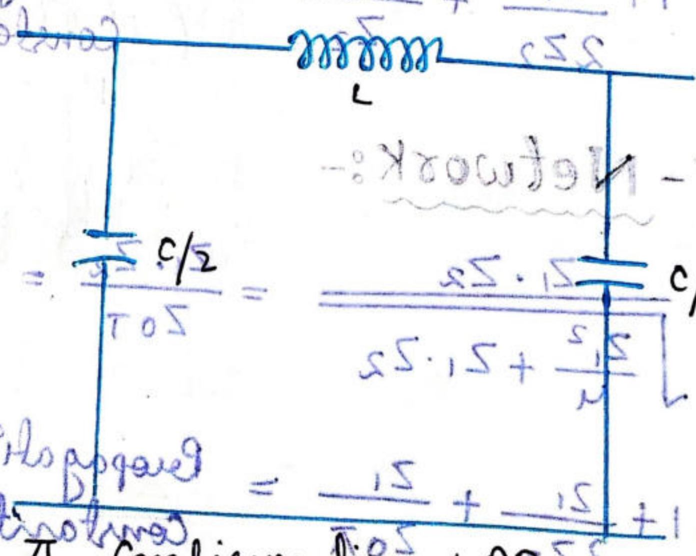


Fig:- π - Configuration LPF

ii) Constant K - high Pass filter (HPF) :-

It is a filter which passes the frequency above cut-off frequency and rejects / attenuates all other frequency below cut-off frequency.

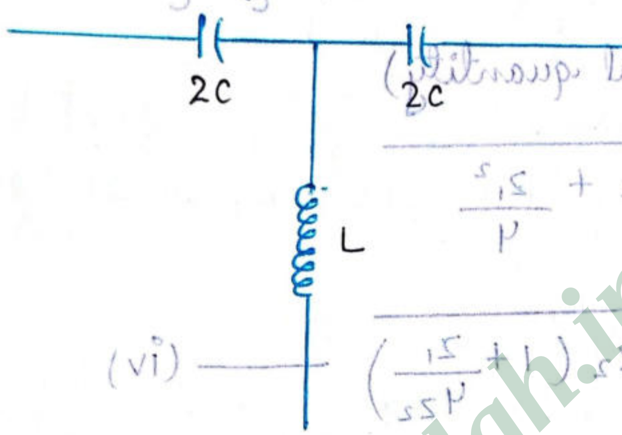


Fig :- T- Configuration of HPF

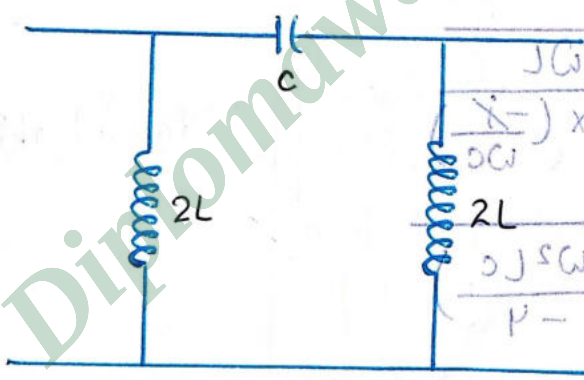


Fig :- π - Configuration of HPF

→ Derivation of LPF :-

Fig. shows T & π section of low Pass filter.

In the 'T' or 'π' section :

Total Series Impedence

$$Z_1 = j\omega L \quad \text{——— ①}$$

$$Z_2 = \frac{1}{j\omega C} = \frac{-j}{\omega C} \quad \text{——— ②}$$

Multiply (i) & (ii)

$$Z_1 \cdot Z_2 = j\omega L \times \frac{-j}{\omega C} = -(j)^2 \frac{L}{C}$$

$$= -(-1) \times \frac{L}{C} = \frac{L}{C} = R_0^2 \quad \text{——— (iii)}$$

($\frac{L}{C}$ is real quantity)

$$Z_{0T} = \sqrt{Z_1 \cdot Z_2 + \frac{Z_1^2}{4}}$$

$$= \sqrt{Z_1 \cdot Z_2 \left(1 + \frac{Z_1}{4Z_2}\right)} \quad \text{——— (iv)}$$

Put $Z_1 \cdot Z_2$ in equation (iv)

$$= \sqrt{\frac{L}{C}\left(1 + \frac{j\omega L}{4 \times \left(\frac{-j}{\omega C}\right)}\right)}$$

$$= \sqrt{\frac{L}{C}\left(1 + \frac{\omega^2 LC}{-4}\right)}$$

$$= \sqrt{\frac{L}{C} \times \left(1 - \frac{\omega^2 LC}{4}\right)}$$

i.e $$Z_{0T} = \sqrt{\frac{L}{C}} \times \sqrt{1 - \frac{\omega^2 LC}{4}}$$

$$Z_{0T} = R_0 \sqrt{1 - \frac{\omega^2 LC}{4}}$$

$$Z_{0T} = R_0 \sqrt{1 - \frac{\omega^2}{\frac{4}{LC}}} \quad \left(\text{where } \omega_c^2 = \frac{4}{LC}\right)$$

$$Z_{0T} = R_0\sqrt{1-\frac{\omega^2}{\omega_c^2}}$$

$$Z_{0T} = R_0\sqrt{1-\frac{(2\pi f)^2}{(2\pi f_c)^2}}$$

$$\boxed{Z_{0T} = R_0\sqrt{1-\left(\frac{f}{f_c}\right)^2}} \quad \text{---} \quad (v)$$

Condition :—

In

$$Z_{0T} = R_0\sqrt{1-\frac{\omega^2 LC}{4}}$$

2f $\frac{\omega^2 LC}{4} < 1$ Z_{0T} will be real.

&

If $\frac{\omega^2 LC}{4} > 1$ Z_{0T} will be Imaginary.

∴ when Z_{0T} is real for Passband.

and Z_{0T} is imaginary for stop band.

Cut-off frequency will be at particular condition $\frac{\omega^2 LC}{4} = 1$

$$\omega^2 LC = 4$$

$$\omega^2 = \frac{4}{LC} \qquad \omega = \sqrt{\frac{4}{LC}}$$

$$\therefore \boxed{\omega = \frac{2}{\sqrt{LC}}}$$

→ Low pass filter has cut-off frequency

$$\omega = \frac{2}{\sqrt{LC}} = 2\pi f = \frac{2}{\sqrt{LC}}$$

$$\boxed{f = \frac{1}{\pi\sqrt{LC}}}$$

→ In the pass band i.e Z_{0T} is real $f < f_c$ and in stop band i.e Z_{0T} is imaginary $f > F_c$.

→ For π Section :-

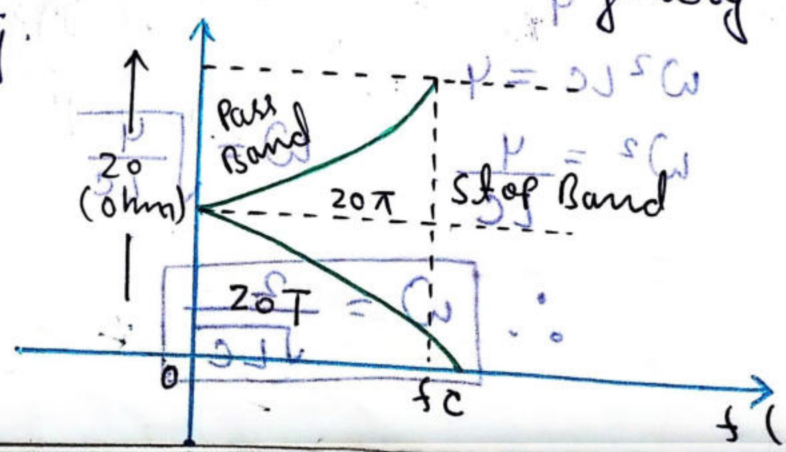
$$Z_{0\pi} = \frac{Z_1 \cdot Z_2}{Z_{0T}} = \frac{R_0^2}{R_0\sqrt{1-\left(\frac{f}{f_c}\right)^2}}$$

$$\boxed{Z_{0\pi} = \frac{R_0}{\sqrt{1-\left(\frac{f}{f_c}\right)^2}}} \text{ —— (vi)}$$

Here,

In Pass band, $f < f_c$, So $Z_{0\pi}$ is real and positive quantity.

when $f > f_c$, $Z_{0\pi}$ becomes imaginary quantity.



→ Derivation of HPF :-

$$Z_1 \cdot Z_2 = R_k^2$$

Assume, Z_1 is the capacitive reactance

$$Z_1 = X_C = \frac{1}{j\omega C} \text{ ——— (i)}$$

Z_2 is the inductive reactance.

$$Z_2 = X_L = j\omega L \text{ ——— (ii)}$$

Multiply the eqn (i) & (ii)

$$Z_1 \cdot Z_2 = \frac{1}{j\omega C} \times j\omega L \qquad \therefore \frac{1}{j} = -j$$

$$= \frac{-j}{\cancel{\omega} C} \times j\cancel{\omega} L$$

$$= \frac{-j^2 L}{C} \qquad \therefore j^2 = -1$$

$$= \frac{-(-1) \cdot L}{C}$$

$$\boxed{Z_1 \cdot Z_2 = \frac{L}{C}}$$

$$Z_1 \cdot Z_2 = R_k^2 = \frac{L}{C}$$

$$R_k = \sqrt{\frac{L}{C}} = R_0 = \frac{\sqrt{L}}{\sqrt{C}} \text{ ——— (iii)}$$

The characteristics impedence of T-network is given by

$$Z_{OT} = \sqrt{\frac{Z_1^2}{4} + Z_1 \cdot Z_2}$$

$$Z_{OT} = \sqrt{\frac{\left(\frac{-j}{\omega C}\right)^2}{4} + \frac{-j}{\cancel{\omega} C} \times j\cancel{\omega} L}$$

$$Z_{OT} = \sqrt{\frac{\frac{(-j)^2}{\omega^2 C^2}}{4} + \frac{L}{C}}$$

$$Z_{OT} = \sqrt{\frac{\frac{-1}{\omega^2 C^2}}{4} + \frac{L}{C}}$$

$$= \sqrt{\frac{L}{C}\left[1 - \frac{1}{4\omega^2 LC}\right]}$$

$$= \sqrt{\frac{L}{C}} \cdot \sqrt{1 - \frac{1}{4\omega^2 LC}}$$

$$\begin{cases} \frac{1}{4\omega^2 LC} < 1 \\ \frac{1}{4\omega^2 LC} > 1 \\ \frac{1}{4\omega^2 LC} = 1 \end{cases}$$

Here, the cut-off frequency is given by

$$\frac{1}{4\omega_c^2 LC} = 1$$

$$1 = 4\omega_c^2 LC$$

$$\frac{1}{4LC} = \omega_c^2$$

(III) —— $$\frac{1}{4LC} = (2\pi f_c)^2$$

$$f_c^2 = \frac{1}{4LC \cdot 4\pi^2}$$

$$f_c^2 = \frac{1}{16\pi^2 LC}$$

$$f_c = \sqrt{\frac{1}{16\pi^2 LC}}$$ $\sqrt{16} = 4$

$\sqrt{\pi^2} = \pi$

$$f_c = \frac{1}{4\pi\sqrt{LC}} \quad \text{--- (IV)}$$

For eqn (III) :-

$$R_0 = \frac{\sqrt{L}}{\sqrt{C}}$$

$$\sqrt{L} = R_0 \cdot \sqrt{C}$$

Substitute '$\sqrt{L}$' into eqn (IV)

$$f_c = \frac{1}{4\pi\sqrt{L} \cdot \sqrt{C}}$$

$$= \frac{1}{4\pi \cdot R_0 \cdot \sqrt{C} \cdot \sqrt{C}}$$

$\sqrt{C} \cdot \sqrt{C} = (\sqrt{C})^2$

$= C.$

$$f_c = \frac{1}{4\pi \cdot R_0 \cdot C}$$

$$\boxed{C = \frac{1}{4\pi R_0 f_c}} \quad \text{--- (V)}$$

For eqn (III)

$$R_0 = \frac{\sqrt{L}}{\sqrt{C}} = \sqrt{C} = \frac{\sqrt{L}}{\sqrt{R_0}}$$

Substitute '$(\sqrt{C})$' in eqn (IV)

$$f_c = \frac{1}{4\pi\sqrt{L}\sqrt{C}}$$

$$= \frac{1}{4\pi\sqrt{L}\cdot\frac{\sqrt{L}}{R_0}}$$

$\sqrt{L}\cdot\sqrt{L} = (\sqrt{L})^2 = L$

$$= \frac{1}{4\pi\frac{L}{R_0}} \quad \text{(VI)}$$

$$f_c = \frac{R_0}{4\pi L}$$

$$\rightarrow F_c = \frac{R_0}{4\pi L}$$

$$\boxed{L = \frac{R_0}{4\pi f_c}} \text{ ——— (VI)}$$