Unit :- 02

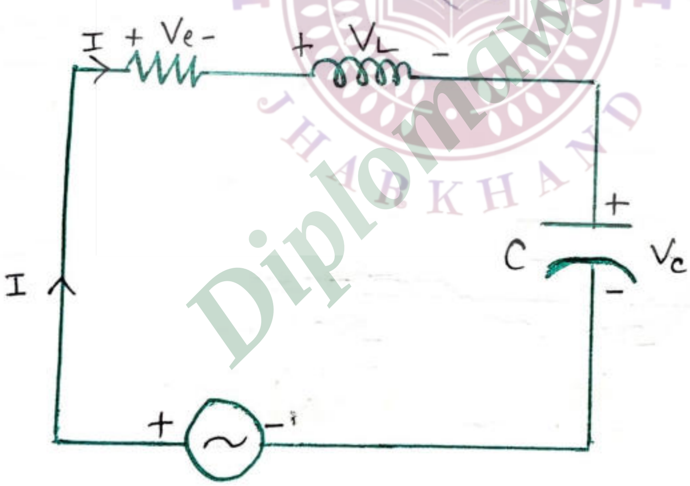
RESONANCE

Date :-   Day :-

*** Resonance :-**

Resonance is the phenomenon at which the response of the circuit is maximum at a given particular frequency.

*** 1) Series Resonance Circuit (RLC) :-**



R → Resistance

L → Inductance

C → Capacitor.

Using KVL (Kirchoff's Voltage law),

$$-V + V_R + V_L + V_c = 0$$

$$V_R + V_L + V_c = V \quad \text{——— (1)}$$

$$V = IR + I(jX_L) + I(-jX_C)$$

↓ Resistance   ↓ Impedence   ↓ Capacitor.

$V = I(R \times jX_L - jX_C)$

$V = I(R + j(X_L - X_C))$

→ **Resonant Condition :-**

when, $X_L - X_C = 0$

$$\boxed{X_L = X_C} \quad \text{—— (11)}$$

For maximum voltage, $[\because X_L = X_C]$

$V = IR + I(j(X_L - X_C))$

$V = IR + I(j(X_L - X_C))$

$$\boxed{V = IR}$$

Here, $X_L = 2\pi f L$

$X_C = \dfrac{1}{2\pi f C}$

$X_L = X_C$

$\therefore \; 2\pi f_r L = \dfrac{1}{2\pi f_r C}$

$f_r^2 = \dfrac{1}{2\pi C \times 2\pi L}$

$f_r^2 = \dfrac{1}{(4\pi)^2 2LC}$

$$f_r = \sqrt{\frac{1}{4\pi^2 LC}}$$

$$\boxed{f_r = \frac{1}{2\pi\sqrt{CL}}} \qquad [f_r = f]$$

Again, by using eqn (i) we get

$$V = V_R + V_L + V_C$$
$$= IR + I(jX_L) + I(-jX_C)$$
$$= I(R + jX_L - jX_C)$$
$$= I(R + j(X_L - X_C))$$

$$\boxed{V = IZ} \text{ — (iii)} \quad [\because V = IR]$$

Here, $Z = (R + j(X_L - X_C))$

Z = Impedence

$$Z = R + j(X_L - X_C)$$
$$Z = R + j \times 0 \qquad [\because X_L = X_C]$$
$$\boxed{Z = R}$$

$$\therefore \boxed{V = IR} \quad [\text{from (iii)}]$$

From eqn — (iii)

Current :-

$$I = \frac{V}{Z}$$

$$I = \frac{V}{R + j(X_L - X_C)}$$

$$I = \frac{V}{R + j \times 0} \quad [\because X_L = X_C]$$

$$\boxed{I = \frac{V}{R}}$$

Proved.

* At resonance, the impedence of series RLC circuit reaches to minimum value. Hence, maximum current flow through this circuit at resonance.

i.e, $\boxed{I = \frac{V}{Z}}$

→ Voltage across resistor (V_R)

$$V_R = IR$$

$$V_R = \left(\frac{V}{\cancel{R}}\right) \times \cancel{R} \quad \left\{ \because I = \frac{V}{R} \right\}$$

$$\boxed{V_R = V}$$

→ Voltage across inductor :- (V_L)

$V_L = I\,(jX_L)$

$V_L = \frac{V}{R}(jX_L)$

$V_L = j\left(\frac{X_L}{R}\right).V$

$$\boxed{V_L = jQV}$$

{Here, Q =

$\because \boxed{Q = \frac{X_L}{R}}$

$\therefore \quad |V_L| = |jQV|$

$$\boxed{|V_L| = QV}$$

→ Voltage across Capacitor (V_C) :-

$V_C = I\,(-jX_C)$

$V_C = \left(\frac{V}{R}\right)(-jX_C)$

$V_C = -j\left(\frac{X_C}{R}\right).V$

$$\boxed{V_C = -jQV}$$

{Q = Quantity Factor}

$$\boxed{Q = \frac{X_C}{R}}$$

$|V_C| = |-jQV| = QV$

* Series resonance RLC circuit is called as voltage magnification circuit because magnitude voltage across the inductor and capacitor is Q times the input sinusoidal voltage (V).

i.e $|V_L| = |V_C| = QV$

Power Factor :-

Power factor = $\cos\theta$ $\because [Z = R]$

$\cos\theta = \frac{R}{Z} = \frac{\cancel{R}}{\cancel{R}} 1$

$\because \cos\theta = 1$

$\theta = \cos^{-1}(1)$

$\boxed{\theta = 0°}$

Resistance ⟶ Same with current

Inductor ⟶ lead

Capacitor ⟶ lag.

* Phasor diagram of series Resonance RLC circuit :-

$$V = V_R + V_L + V_C$$

$$\bar{V} = \bar{V}_R + \bar{V}_L + \bar{V}_C \quad \text{[Phasor]}$$

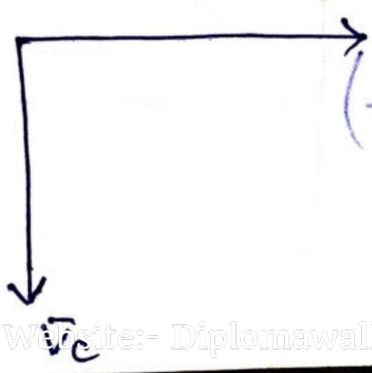
$$\bar{I}Z = \bar{I}R + \bar{I}(j\bar{X}_L) + \bar{I}(-j\bar{X}_C)$$

→ Phasor diagram :-

① Resistance :-

(II) Inductor :- $(\bar{V}_L)$

(III) Capacitor :- $(\bar{V}_C)$



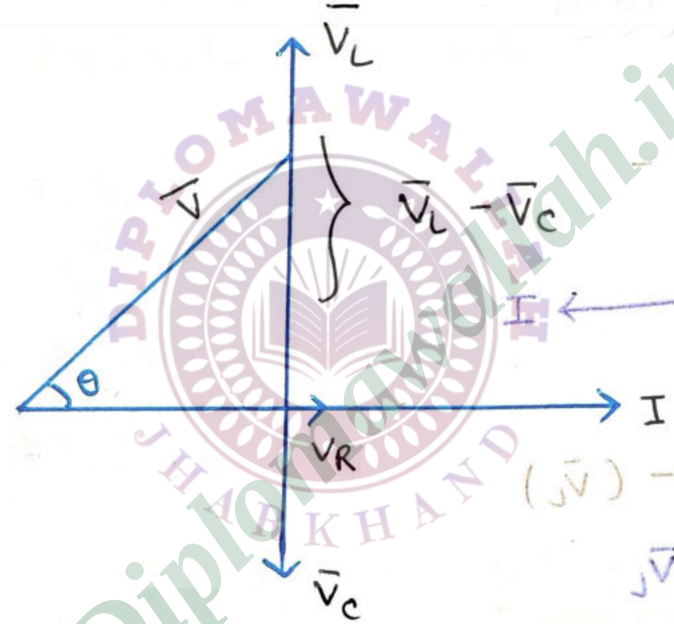
* For Voltage triangle phasor diagram

Assum $(V_L > V_C)$

Using Pythagoras theorem,

$$\bar{V}^2 = \bar{V}_R^2 + (\bar{V}_L - \bar{V}_C)^2$$

$$\bar{V} = \sqrt{\bar{V}_R^2 + (\bar{V}_L + \bar{V}_C)^2}$$



* Power factor Angle :-

$$\tan\theta = \frac{P}{b} = \frac{\bar{V}_L - \bar{V}_C}{\bar{V}_R}$$

$$\boxed{\theta = \tan^{-1}\left(\frac{\bar{V}_L - \bar{V}_C}{\bar{V}_R}\right)}$$

* Power factor :-

$$\cos\theta = \frac{b}{h} = \frac{\bar{V}_R}{\bar{V}}$$

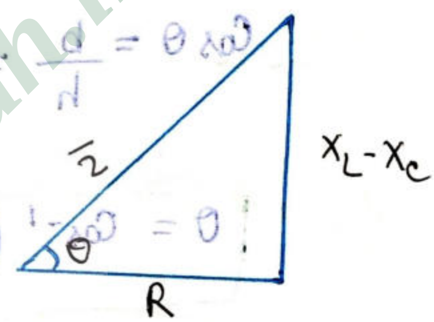
$$\boxed{\theta = \cos^{-1}\left(\frac{\bar{V}_R}{\bar{V}}\right)}$$

For impedence triangle phasor diagram.

* Power factor angle :-

$$\tan\theta = \frac{p}{b} = \frac{\bar{X}_L - \bar{X}_C}{\bar{R}}$$

$$\boxed{\theta = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)}$$



* Power factor :-

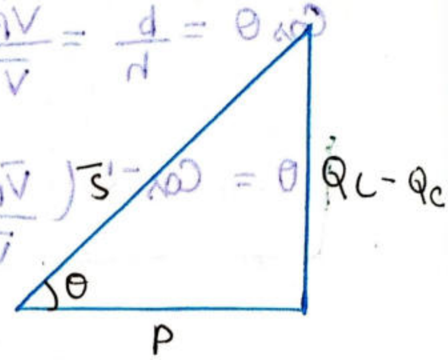
$$\cos\theta = \frac{b}{h} = \frac{R}{Z}$$

$$\boxed{\theta = \cos^{-1}\frac{R}{Z}}$$

* Power Triangle :-

→ Power factor angle

$$\tan\theta = \frac{P}{b} = \frac{Q_L - Q_C}{P}$$



$$\boxed{\theta = \tan^{-1}\left(\frac{Q_L - Q_C}{P}\right)}$$

→ Power factor :-

$$\cos\theta = \frac{b}{h} = \frac{P}{S}$$

$$\boxed{\theta = \cos^{-1}\left(\frac{P}{S}\right)}$$

Characteristics of Series Resonance Circuit :-

- Resonance occur when $X_L = X_C$.
- Resonant frequency $f_r = \frac{1}{2\pi\sqrt{LC}}$
- Impedence is minimum and the circuit is purely resistive.
- Current has a maximum value.

- When a number of frequencies are fed to it, it accept only one frequency i.e, resonant frequency (f_r) and reject all other frequencies.
- The current is maximum for this frequency and hence it is called accepter circuit.

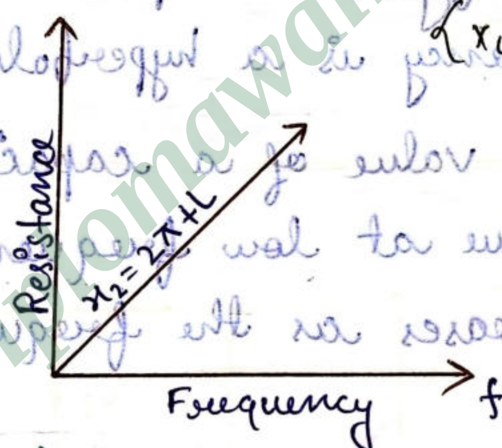
Resonance Plot

→ **Resonance plot for Inductor :-**

Plot -1

$\{x_L, x_C = \text{reactance}\}$

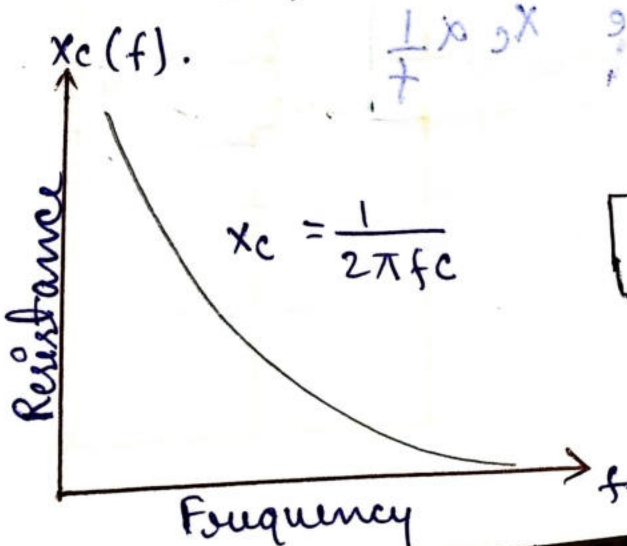
(A)



$\boxed{x_L \propto f}$

→ **Resonant Plot for Capacitor :-**

Plot -2



$\boxed{X_C \propto \frac{1}{f}}$

Plot : 1 :-

The graph in fig(A) is inductive reactance against frequency is a straight line linear curve. The inductive reactance value of an inductor increases linearly as the frequency increase.

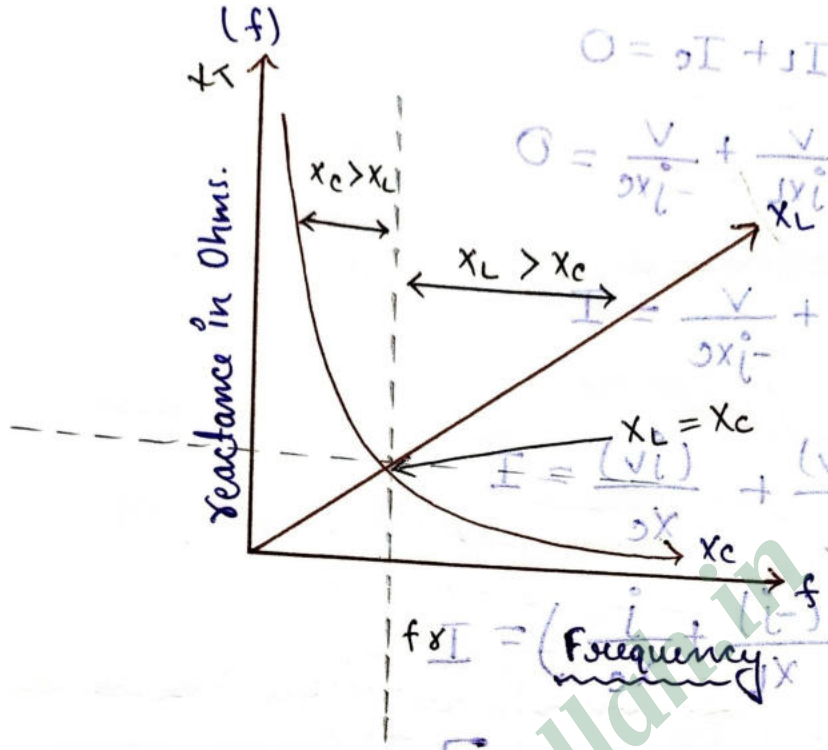
i.e $\boxed{X_L \alpha f}$

Plot : 2 :-

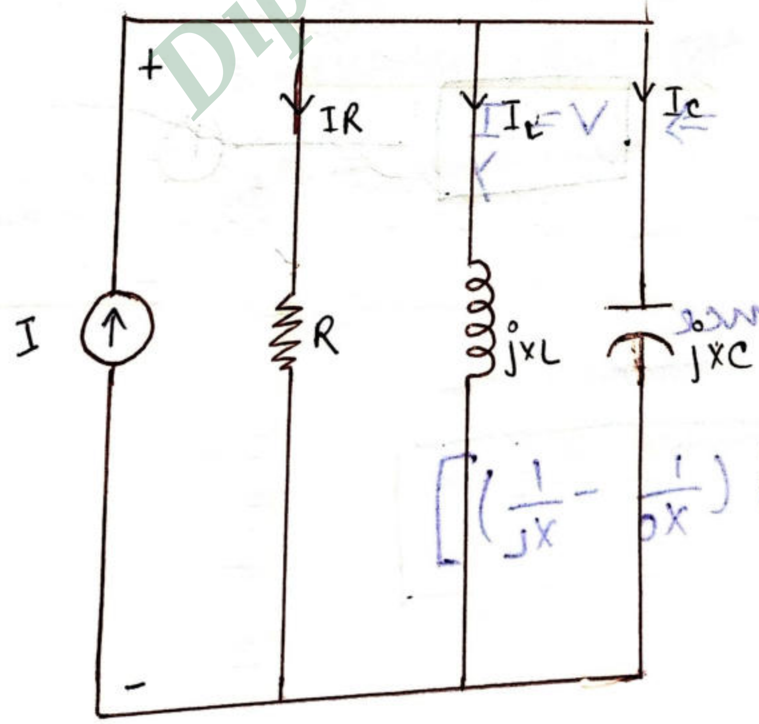
The graph in fig(B) is capacitive reactance against frequency is a hyperbolic curve. The reactance value of a capacitor has a very high value at low frequency but quickly decreases as the frequency increases.

$\boxed{\text{i.e} \quad X_C \alpha \frac{1}{f}}$

* Series resonant Plot :-



2.) Parallel Resonance Circuit



Using KCL,

$$-I + I_R + I_L + I_C = 0$$

$$-I + \frac{V}{R} + \frac{V}{jX_L} + \frac{V}{-jX_C} = 0$$

$$\frac{V}{R} + \frac{V}{jX_L} + \frac{V}{-jX_C} = I$$

$$\frac{V}{R} + \frac{(-jV)}{X_L} + \frac{(jV)}{X_C} = I$$

$$V\left(\frac{1}{R} + \frac{(-j)}{X_L} + \frac{j}{X_C}\right) = I$$

$$V\left[\frac{1}{R} + (j)\left(\frac{-1}{X_L} + \frac{1}{X_C}\right)\right] = I.$$

$$V\left[\frac{1}{R} + j\left(\frac{1}{X_C} - \frac{1}{X_L}\right)\right] = I$$

$$\boxed{I = VY} \Rightarrow \boxed{V = \frac{I}{Y}} \quad — \quad (1)$$

Y = Admitance

$$\boxed{Y = \left[\frac{1}{R} + j\left(\frac{1}{X_C} - \frac{1}{X_L}\right)\right]}$$

→ ∴ The admintance 'Y' of the parallel RLC circuit will be

$$\boxed{Y = \frac{1}{R} + j\left(\frac{1}{X_C} - \frac{1}{X_L}\right)}$$

→ In Parallel RLC circuit resonance occur when imaginary term of admintance, 'Y' is 0.

i.e the value of $\boxed{\frac{1}{X_C} - \frac{1}{X_L} = 0}$

$$\frac{1}{X_C} = \frac{1}{X_L}$$

$$\boxed{X_L = X_C}$$

The above resonance condition is same as that of series RLC circuit. So, the resonance frequency, f_r will be same in both series RLC circuit and parallel RLC circuit.

$$\boxed{f_r = \frac{1}{2\pi\sqrt{LC}}}$$

* **Admitance :-**

$$Y = \frac{1}{R} + j\left(\frac{1}{X_C} - \frac{1}{X_L}\right)$$

Now, $X_L = X_C$

$$Y = \frac{1}{R} + j(0)$$

$$\boxed{Y = \frac{1}{R}}$$

∵ The admintance 'Y' of parallel RLC circuit is equal to the reciprocal of resistor 'R'.

i.e $\boxed{Y = \frac{1}{R}}$

* Voltage :- Voltage across each element.

∵ $\boxed{\frac{1}{X_C} - \frac{1}{X_L} = 0}$

$$I = V\left[\frac{1}{R} + j(0)\right]$$

$$I = \frac{V}{R}$$

$$\boxed{V = IR}$$

$\because$ Voltage across all the element of parallel RLC circuit at resonance $V = IR$

The admitance of Parallel RLC circuit reaches its minimum value hence, maximum voltage is present across each element of this circuit at resonance.

$$\boxed{V\uparrow = \frac{I}{Y\downarrow}} \quad [\text{from (ii)}]$$

* Current flowing through Resistor :- (I_R)

$$I_R = \frac{V}{R}$$

$$I_R = \frac{I\cancel{R}}{\cancel{R}} \quad \{\text{using ohm's law}\}$$

$$\boxed{I_R = I}$$

$\therefore$ Current flowing through resistor at resonance

$$\boxed{I_R = I}$$

* Current flowing through Inductor

$$I_L = \frac{V}{jX_L}$$

$$I_L = \frac{IR}{jX_L}$$

$$I_L = \frac{-jIR}{X_L}$$

$$I_L = -j\left(\frac{R}{X_L}\right) \times I$$

$$\boxed{I_L = -jQI} \qquad \boxed{Q = \frac{R}{X_L}}$$

Q = quality factor.

Magnitude

$$\boxed{|I_L| = |jQI|}$$

$$\boxed{\frac{I}{Y} = V}$$

* Current flowing through Capacitor :-

$$I_c = \frac{V}{-jX_C}$$

$$I_c = j\frac{IR}{X_c}$$

$$I_c = j\left(\frac{R}{X_c}\right) I.$$

$$\boxed{I_c = jQI}$$

Q = Quality Factor $\left\{Q = \frac{R}{X_c}\right\}$

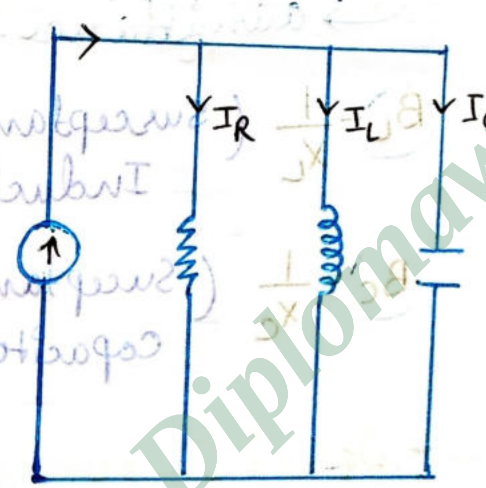
$$\boxed{Q = \frac{R}{X_c}}$$

$$\boxed{|I_c| = |QI|}$$ { Magnitude }

∴ Parallel resonance RLC circuit is called as current ~~magnitude~~ magnification circuit because magnitude of the current flowing through inductor and capacitance is equal to Q the input sinusoidal current 'I'.

Date :- 09.12.24 Day :- Monday

Phasor Diagram



$$\bar{I} = \bar{I}_R + \bar{I}_L + \bar{I}_C$$

$$\frac{\bar{V}}{\bar{Z}} = \frac{V}{R} + \frac{V}{jX_L} + \frac{V}{-jX_C}$$

$$\frac{\cancel{V}}{Z} = \cancel{V}\left(\frac{1}{R} + \frac{1}{jX_L} + \frac{V}{-jX_C}\right)$$

$$\frac{1}{Z} = \left(\frac{1}{R} + \frac{1}{jX_L} - \frac{1}{jX_C}\right)$$

$$\frac{1}{Z} = \left(\frac{1}{R} - \frac{j}{X_L} + \frac{j}{X_C}\right)$$

$$\frac{1}{Z} = \left(\frac{1}{R} + j\left(\frac{1}{X_C} - \frac{1}{X_L}\right)\right)$$

$$\boxed{Y = \{G + j(B_C - B_L)\}}$$

R = Resistance
G = Conductance
Y = Admittance
Z = Impedence
L = Inductance
C = Capacitance.

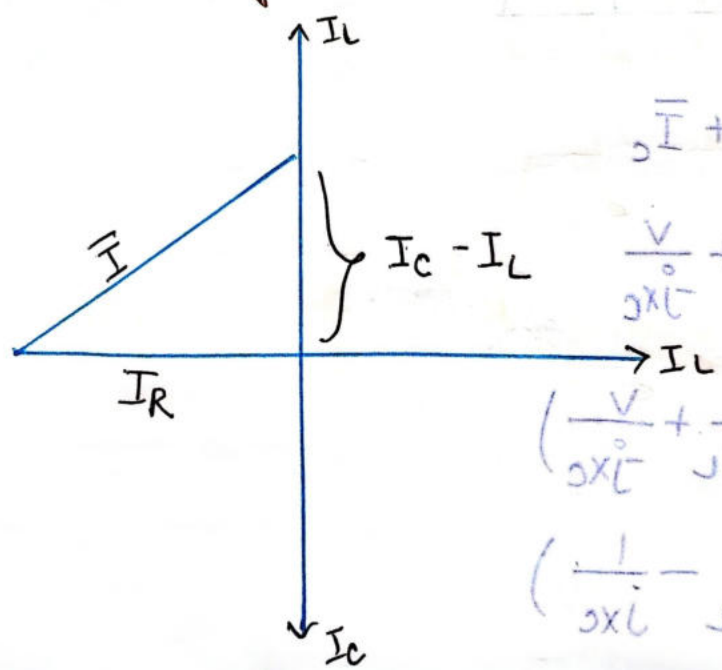
$G = \frac{1}{R}$ (Reciprocal of resistance) → Conductance

$Y = \frac{1}{Z}$ (Reciprocal of Impedence) → admittance.

$B_L = \frac{1}{X_L}$ (Susceptance of Inductor)

$B_C = \frac{1}{X_C}$ (Suceptance of capacitor)

★ Current Triangle :-



By using Pythogoras theorems

$$h^2 = p^2 + b^2$$

$$\bar{I}^2 = \bar{I}_R^2 + (\bar{I}_C - \bar{I}_L)^2$$

$$\bar{I} = \sqrt{\bar{I}_R^2 + (I_C - I_L)^2}$$

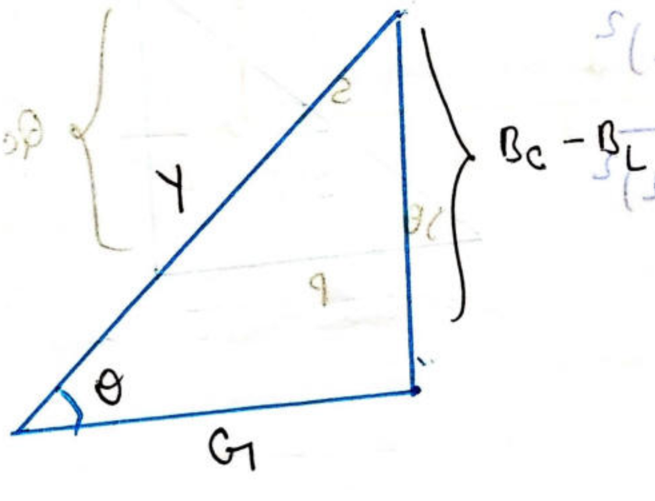
* Power factor angle $\tan\theta = \frac{P}{b} = \frac{\bar{I}_C - \bar{I}_L}{\bar{I}_R}$

$$\boxed{\theta = \tan^{-1}\left(\frac{\bar{I}_C - \bar{I}_L}{\bar{I}_R}\right)}$$

* Power factor $\cos\theta = \frac{b}{h} = \frac{\bar{I}_R}{\bar{I}}$

$$\boxed{\theta = \cos^{-1}\left(\frac{\bar{I}_R}{\bar{I}}\right)}$$

Admittance triangle :-



By using Pythagoras theorem :-

$$h^2 = p^2 + b^2$$

$$\bar{Y}^2 = G^2 + (B_C - B_L)^2$$

$$\bar{Y} = \sqrt{G^2 + (B_C - B_L)^2}$$

* Power factor angle $\tan\theta = \frac{p}{b} = \frac{B_C - B_L}{G}$

$$\boxed{\theta = \tan^{-1}\left(\frac{B_C - B_L}{G}\right)}$$

* Power factor $\cos\theta = \frac{b}{h} = \frac{G}{Y}$

$$\boxed{\theta = \cos^{-1}\left(\frac{G}{Y}\right)}$$

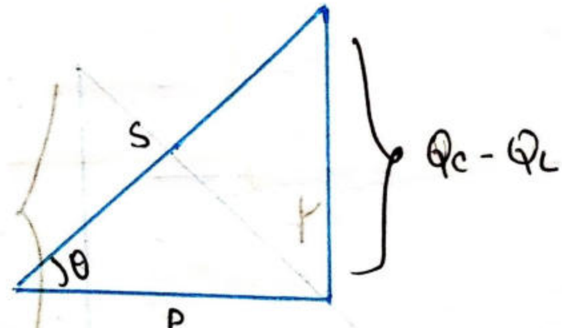
Power triangle :-

By using Pythagoras theorem :-

$$h^2 = p^2 + b^2$$

$$S^2 = P^2 + (Q_C - Q_L)^2$$

$$\bar{S} = \sqrt{P^2 + (Q_C - Q_L)^2}$$



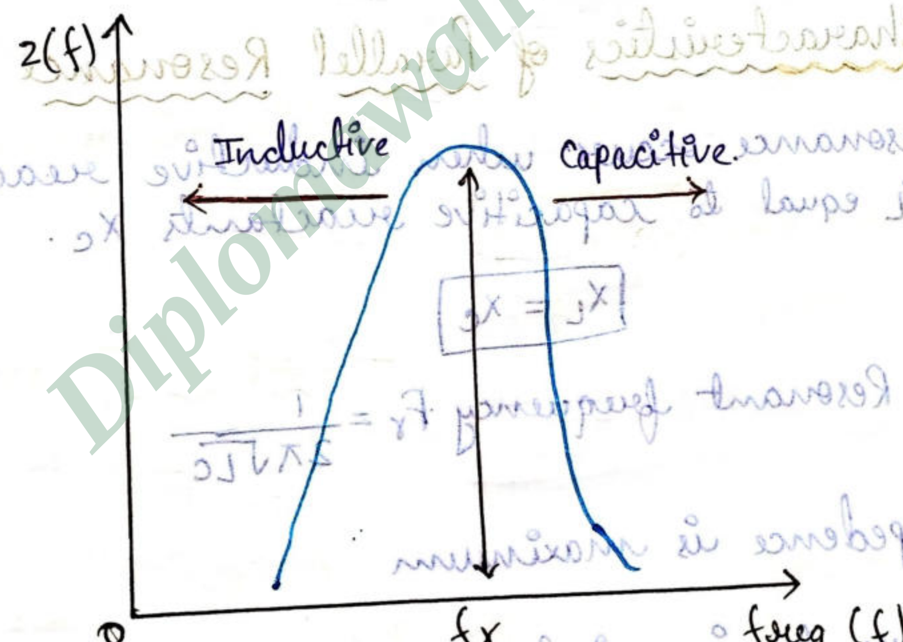
* Power factor angle $\tan\theta = \frac{P}{b} = \frac{Q_C - Q_L}{P}$

$$\boxed{\theta = \tan^{-1}\left(\frac{Q_C - Q_L}{P}\right)}$$

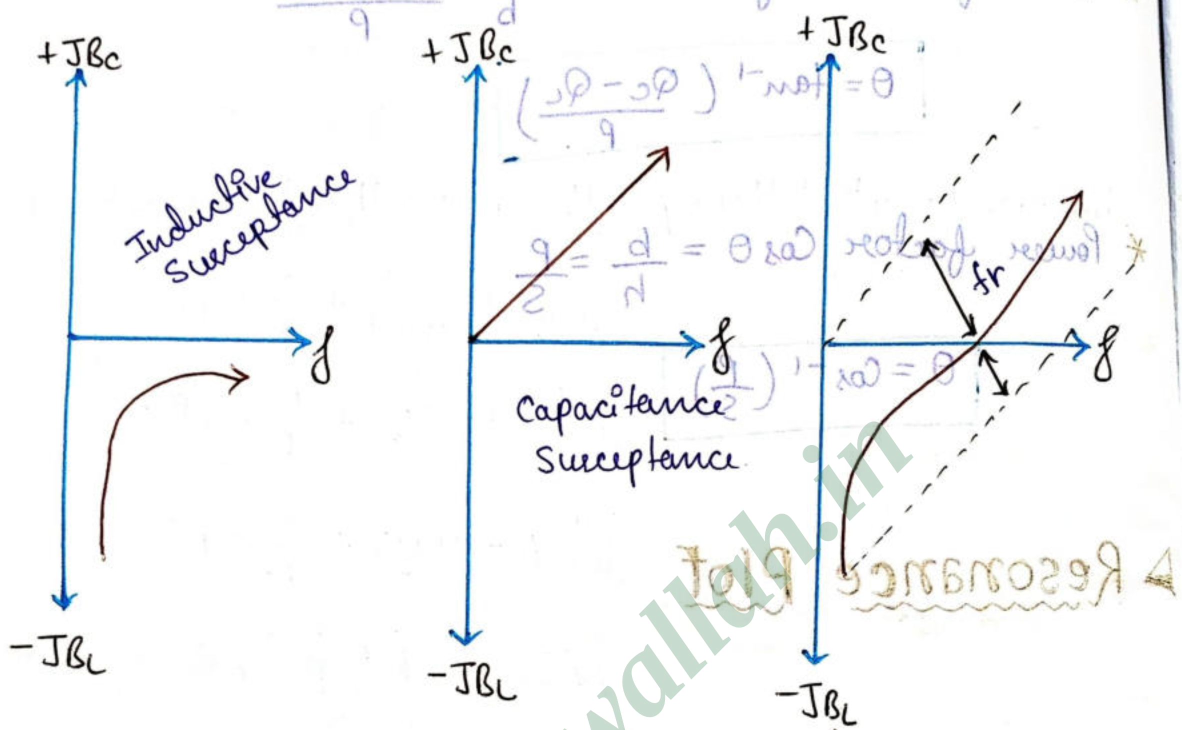
* Power factor $\cos\theta = \frac{b}{h} = \frac{P}{S}$

$$\boxed{\theta = \cos^{-1}\left(\frac{P}{S}\right)}$$

▷ Resonance Plot



Susceptance at Resonance



Characteristics of Parallel Resonance

1) Resonance occurs when inductive reactants X_L equal to capacitive reactants X_C.

$$\boxed{X_L = X_C}$$

Resonant frequency $F_r = \frac{1}{2\pi\sqrt{LC}}$

2) Impedence is maximum

3) Current is minimium.

4) The circuit rejects F_r but allows the current to flow for other frequencies. Hence, it is called a rejector circuit.

* Condition for resonance parallel RLC circuit :-

- In parallel resonance circuit the condition for resonance occurs when the reactive power of the capacitor cancel each other out.
- This typically happens at a specific frequency known as resonant frequency.

* Condition :-

There are two types of condition :-

① Impedence condition :-

The total impedence of the circuit becomes purely resistive at resonant frequency.

② Frequency Condition :-

The resonant frequency can be calculated using the formula $f_r = \frac{1}{2\pi\sqrt{LC}}$

where as L is the inductance in henries C is the capacitance in Farad.

Date:- 13.12.24 Day:- Friday

Expression for resonant frequency for Parallel RLC circuit

The total admittance of the parallel tuned circuit.

$$Y_T = \frac{1}{R} + \frac{1}{1/j\omega C} + \frac{1}{j\omega L}$$

$$Y_T = \frac{1}{R} + j\omega C + \frac{1}{j\omega L}$$

$$Y_T = \frac{1}{R} + j\omega C - j\frac{1}{\omega L}$$

$$Y_T = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)$$

At resonant condition the imaginary part becomes zero.

$$\therefore \boxed{\omega C = \frac{1}{\omega L}}$$

$$\omega^2 = \frac{1}{LC}$$

$$\boxed{\omega = \frac{1}{\sqrt{LC}}}$$

$$\omega = 2\pi f$$

$$\omega = \frac{1}{\sqrt{LC}}$$

$$2\pi f = \frac{1}{\sqrt{LC}}$$

$$\boxed{f = \frac{1}{2\pi\sqrt{LC}}}$$

$\left\{ \begin{array}{l} \omega = 2\pi f \\ \text{angular frequency} \end{array} \right.$

Series Resonance :

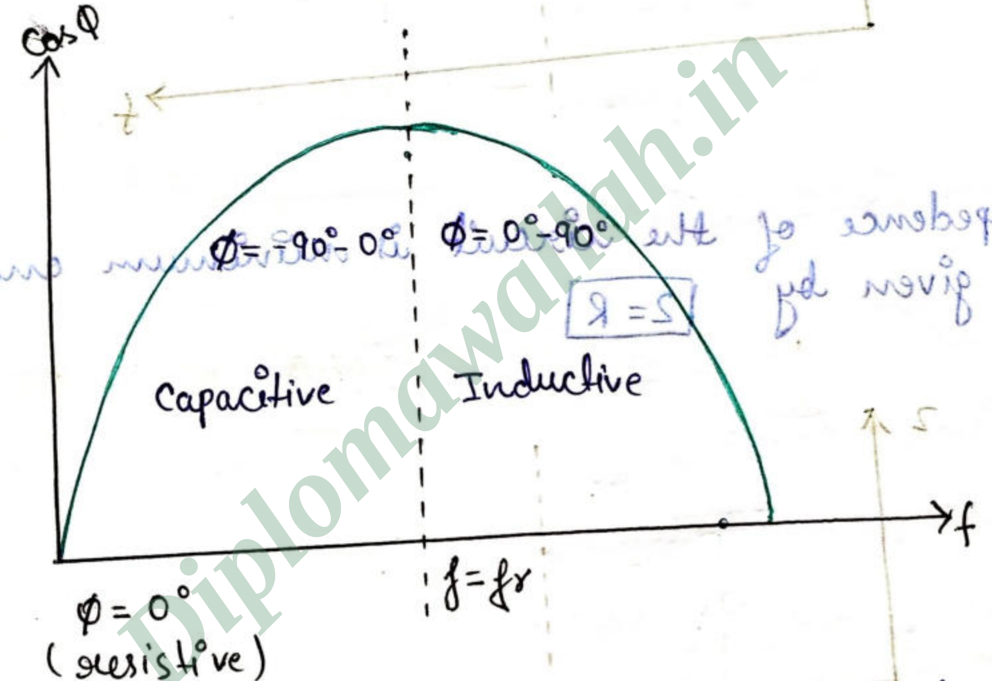
\# Condition for RLC series circuit :-

i) Inductive reactants should be equal to capacitative reactants.

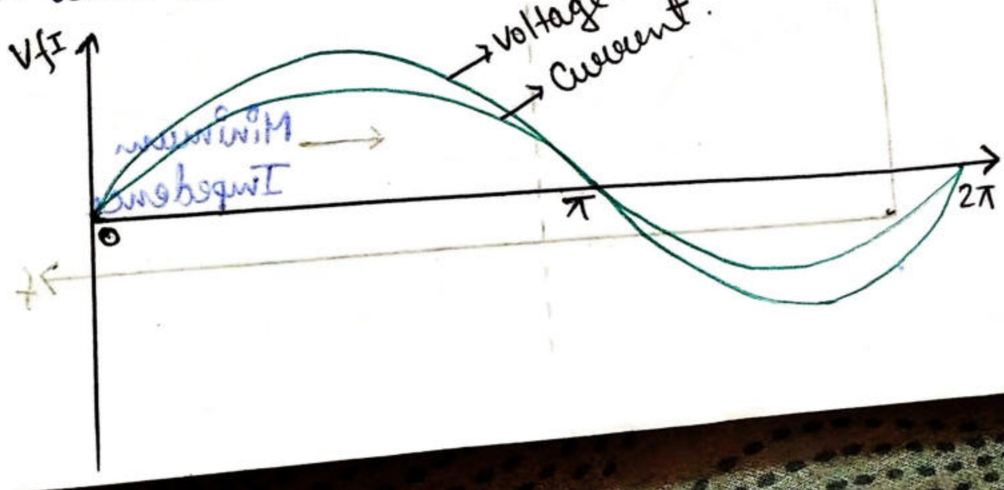
$$\boxed{X_L = X_C}$$

ii) The power factor Cos ϕ is equal to 1. where $\phi = 0$

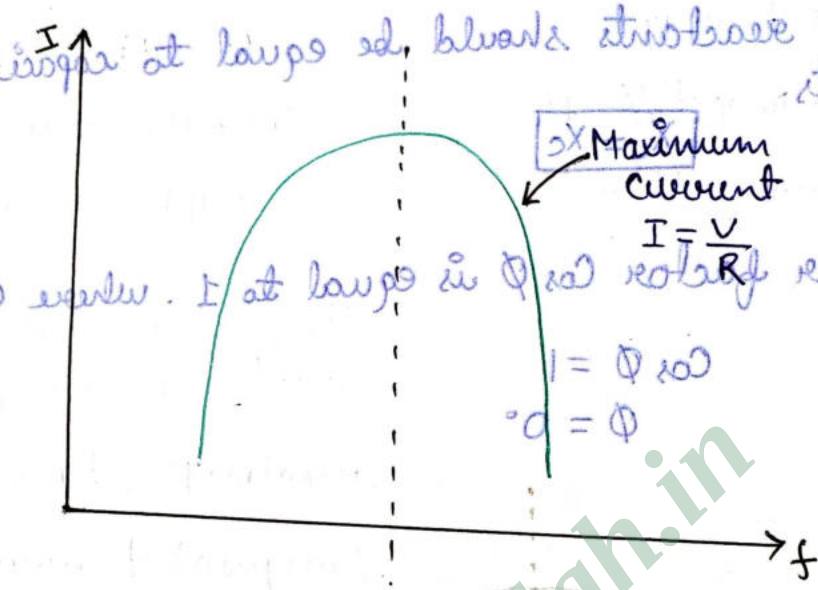
$$\cos\phi = 1$$
$$\phi = 0°$$



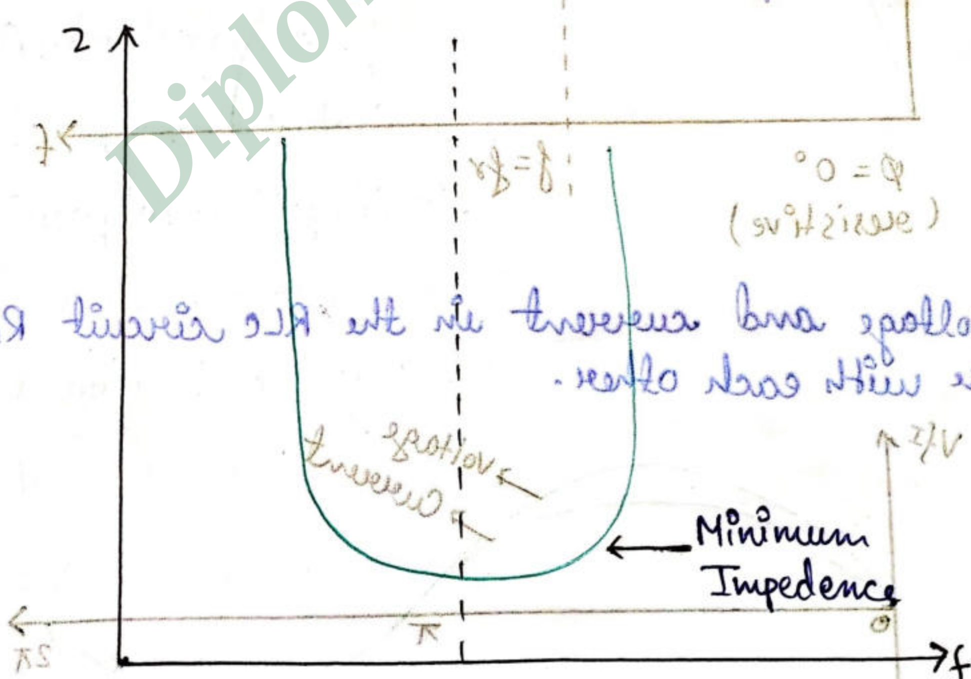
iii) The voltage and current in the RLC circuit Rin phase with each other.



iv) Current in the circuit is maximum and given by $\boxed{I = \frac{V}{R}}$



v) Impedence of the circuit is minimum and is given by $\boxed{Z = R}$



Expression for Resonant frequency in series RLC circuit.

Inductive reactants = Capacitive reactants.

$$X_L = X_C$$

$$\omega_L = \frac{1}{\omega_C}$$

$$2\pi f L = \frac{1}{2\pi f C}$$

$$f^2 = \frac{1}{(2\pi)^2 LC}$$

$$f = \frac{1}{\sqrt{(2\pi^2)LC}}$$

$$\boxed{f = \frac{1}{2\pi\sqrt{LC}}}$$

Symbol with Names

R = Resistor.

C = Capacitor (Farad)

L = Inductor (Henry).

f = Frequency (Hz)

f_r = Resonant frequency.

X_C = Capacitive Reactance.

X_L = Inductive Reactance.

I = Current (ampere).

V = Voltage (Volt).

Z = Impedance (Ω)

Y = Admittance = $Y = \frac{1}{Z}$

G = Conductance = $G = \frac{1}{R}$.

B_L = Susceptance (Inductor).

B_C = Susceptance (Capacitor).

Q = Quality factor.

ω = Angular frequency $\omega = 2\pi f$

$\cos\phi$ = Power Factor.

$\tan^{-1}()$ = ϕ. Power factor angle.

j = Imaginary.

| | = Mode/Magnitude.

P = Total Power

S = Apparent Power

V_R = Voltage across resistor.

V_L = Voltage across Inductor.

V_C = Voltage across Capacitor.