Unit :- 02

Statistical Analysis...

Arithmetic Mean :-

The arithmetic mean is the sum of a set of numbers divided by the number of elements in the set. It is commonly reffered to as the "average".

* Formula :-

$$\boxed{\text{Arithmetic Mean} = \frac{\text{Sum of all values}}{\text{Number of Values.}}}$$

- Σx_i = Sum of all data values.
- n = Total number of Values.

* Example :-

Consider the number : 4, 7, 9

1. Sum of Values : $4+7+9 = 20$
2. No. of Values : $n = 3$

$$\text{Arithmetic Mean} = \frac{20}{3} = 6.67$$

So, the arithmetic mean is 6.67.

Deviation :-

Deviation is the difference between a data value and a central value, typically the mean or median of the dataset. It shows how far a specific data point is from the central tendency.

* **Formula :-**

The deviation of a data point x_i from the mean Mean:

$$\boxed{\text{Deviation} = x_i - \text{Mean}}$$

* **Example :-**

Consider the dataset : 4, 7, 9.

① Find the mean :

$$\text{Mean} = \frac{4+7+9}{3} = \frac{20}{3} = 6.67$$

1. Calculate the deviations :

- For $x_1 = 4$

$4 - 6.67 = -2.67$

- For $x_2 = 7$

$7 - 6.67 = 0.33$

- For x_3

$9 - 6.67 = 2.33$

So, the deviations for the dataset are -2.67, 0.33 and 2.33

Average Deviation :-

Average deviation is the average of the absolute deviations of each data value from a central value, usually the mean or median. It measures how much, on average, data points differ from the central value.

* Formula :-

For a dataset with values

$x_1, x_2, \ldots\ldots, x_n$:

$$\boxed{\text{Average Deviation} = \frac{\Sigma |x_i - c|}{n}}$$

* Example :-

Consider the dataset : 4, 7, 9

① Find the Mean :

$$\text{Mean} = \frac{4+7+9}{3} = 6.67$$

1) Calculate absolute deviations :-

- $|4 - 6.67| = 2.67$
- $|7 - 6.67| = 0.33$
- $|9 - 6.67| = 2.33$

2) Find the average deviations :-

$$\text{Average Deviation} = \frac{2.67 + 0.33 + 2.33}{3} = 1.78$$

So, the average deviation is 1.78

Standard Deviation :-

Standard deviation is a measure of the dispersion or spread of data points around the mean. It shows how much the values in a dataset vary from the average value. A smaller standard deviation indicates that data points are closer to the mean, while a larger standard deviation indicates more spread.

* Formula :-

For a dataset with values

$x_1, x_2, \ldots, x_n$:

Population standard Deviation :-

$$\boxed{\sigma = \sqrt{\frac{\Sigma (x_i - \mu)^2}{N}}}$$

Sample standard Deviation :-

$$\boxed{S = \sqrt{\frac{\Sigma (x_i - \bar{x})^2}{n-1}}}$$

Where :

- x_i = Individual data Points.
- μ = Population mean, $\bar{x}$ = Sample Mean.
- N = Total number of values (for population)
- n = Total number of values (for sample)

* **Example :-**

Consider the dataset : 4, 7, 9 (Sample)

① Find the mean :

$$\text{Mean} = \frac{4+7+9}{3} = \frac{20}{3} = 6.67$$

1.) Calculate squared deviations from the mean:

- $(4-6.67)^2 = 7.11$
- $(7-6.67)^2 = 0.11$
- $(9-6.67)^2 = 5.44$

2.) Sum of squared deviations :-

$$\Sigma (x_i - \bar{x})^2 = 7.11 + 0.11 + 5.44 = 12.66$$

∴ Calculate the standard deviation :

$$s = \sqrt{\frac{12.66}{3-1}} = \sqrt{6.33}$$

$$= 2.52$$

So, the sample standard deviation is 2.52.

Probability of errors :-

The Probability of errors refers to the likelihood that an error or incorrect outcome occurs in a given process, system, or experiment. It is a statistical measure used in various fields like quality control, communication systems, and hypothesis testing to assess the frequency or chance of errors under specific conditions.

* Formula (General formula)

The Probability of error is calculated as:

$$\boxed{P(\text{Error}) = \frac{\text{Number of Errors}}{\text{Total Number of Trials}}}$$

* Example :-

Suppose in a signal transmission, 5 errors occurred out of 1,000 transmitted bits.

1.) Calculate the Probability of error:-

$$P(\text{Error}) = \frac{5}{1000} = 0.005$$

So, the probability of error is 0.005 or 0.5%

* Limiting Error :-

The limiting error (also called the maximum possible error) is the maximum deviation of a measured or calculated value from its true or nominal value, as specified by the manufacturer or determined by the conditions of measurement. It is quantity the worst-case error in a measurement system.

* Formula for limiting error :-

$$\boxed{\text{Limiting error} = \frac{\text{Maximum Deviation}}{\text{Nominal Value}} \times 100\% \text{ (if expressed as a percentage)}}$$

* Example :-

Given :

- Nominal Value of a resistor : $100\,\Omega$
- Tolerance ± 5

Solution :- The maximum possibe deviation is ± 5 of $100\,\Omega$.

$$\text{Maximum Deviation} = \frac{5}{100} \times 100\,\Omega = \pm 5\,\Omega$$

Thus, the limiting error is $\pm 5\,\Omega$

▶ Problems on Statistical Analysis :-

Date :- 03.12.24 Day :- Tuesday

Given,

$x_1 = 99.7$

$x_2 = 100.1$

$x_3 = 100.2$

$x_4 = 99.6$

$x_5 = 99.7$

Q.1 Find the arithmetic mean, average deviation, standard deviation, deviation of each value and algebraic sum of the deviation.

1) Arithmetic Mean :-

$$= \frac{99.7 + 100.1 + 100.2 + 99.6 + 99.7}{5}$$

$$= \frac{499.3}{5}$$

$$= \boxed{99.86}$$

2) Deviation of each value :-

$d_1 = x_1 - \bar{x} = 99.7 - 99.86 = -0.16$

$d_2 = x_2 - \bar{x} = 100.1 - 99.86 = 0.24$

$d_3 = x_3 - \bar{x} = 100.2 - 99.86 = 0.34$

$d_4 = x_4 - \bar{x} = 99.6 - 99.86 = -0.26$

$d_5 = x_5 - \bar{x} = 99.7 - 99.86 = -0.16$

3) Algebraic Sum of deviation :-

$$d = -0.16 + 0.24 + 0.34 - 0.26 - 0.16$$

$$= 0.58 - 0.58$$

$$\boxed{d = 0}$$

4) Average deviation :-

$$D = \frac{|d_1| + |d_2| + |d_3| + |d_4| + |d_5|}{5}$$

$$= \frac{0.16 + 0.24 + 0.34 + 0.26 + 0.16}{5}$$

$$= \frac{1.16}{5}$$

$$\boxed{D = 0.232}$$

5) Standard Deviation :-

$$6 = \sqrt{\frac{d_1^2 + d_2^2 + d_3^2 + d_4^2 + d_5^2}{n-1}}$$

$$= \sqrt{\frac{(-0.16)^2 + (0.24)^2 + (0.34)^2 + (-0.26)^2 + (-0.16)^2}{5-1}}$$

$$= \sqrt{\frac{0.292}{4}}$$

$$\boxed{6 = 0.27}$$

Q.2 Given, Date:- 04/12/24 Wednesday

$x_1 = 51.2\,\Omega$

$x_2 = 51.7\,\Omega$

$x_3 = 51.3\,\Omega$

$x_4 = 51\,\Omega$

$x_5 = 51.5\,\Omega$

$x_6 = 51.3\,\Omega$

$x_7 = 51.2\,\Omega$

$x_8 = 51.4\,\Omega$

$x_9 = 51.3\,\Omega$

$x_{10} = 51.1\,\Omega$

Q: Find the arithmetic mean, the standard deviation, average deviation and probable error.

1.) The arithmetic Mean:-

$$\bar{x} = \frac{\Sigma x}{n}$$

$$\bar{x} = \frac{51.2 + 51.7 + 51.3 + 51 + 51.5 + 51.3 + 51.2 + 51.4 + 51.3 + 51.1}{10}$$

$$= \frac{513}{10}$$

$$\boxed{\bar{x} = 51.3}$$

2) Deviation of each value :-

$$d_1 = x_1 - \bar{x} = 51.2 - 51.3 = -0.1$$

$$d_2 = x_2 - \bar{x} = 51.7 - 51.3 = 0.4$$

$$d_3 = x_3 - \bar{x} = 51.3 - 51.3 = 0$$

$$d_4 = x_4 - \bar{x} = 51. - 51.3 = -0.3$$

$$d_5 = x_5 - \bar{x} = 51.5 - 51.3 = 0.2$$

$$d_6 = x_6 - \bar{x} = 51.3 - 51.3 = 0$$

$$d_7 = x_7 - \bar{x} = 51.2 - 51.3 = -0.1$$

$$d_8 = x_8 - \bar{x} = 51.4 - 51.3 = 0.1$$

$$d_9 = x_9 - \bar{x} = 51.3 - 51.3 = 0$$

$$d_{10} = x_{10} - \bar{x} = 51.1 - 51.3 = -0.2$$

3) Standard deviation :-

$$\sigma = \sqrt{\frac{d_1^2 + d_2^2 + d_3^2 + d_4^2 + d_5^2 + d_6^2 + d_7^2 + d_8^2 + d_9^2 + d_{10}^2}{n-1}}$$

$$= \sqrt{\frac{(-0.1)^2 + (0.4)^2 + (0)^2 + (-0.3)^2 + (0.2)^2 + (0)^2 + (-0.1)^2 + (0.1)^2 + (0)^2 + (-}{10-1}}$$

$$= \sqrt{\frac{(-0.1)^2 + (0.4)^2 + (-0.3)^2 + \cancel{(0.2)^2} + \cancel{(-0.1)} + \cancel{(0.1)} + (-0.2)^2}{9}}$$

$$= \sqrt{\frac{(-0.1)^2 + (0.4)^2 + (-0.3)^2}{9}}$$

$$= \sqrt{\frac{0.36}{9}} = \sqrt{0.04}$$

$$\boxed{\sigma = 0.2\ \Omega}$$

4.) Average Deviation :-

$$= \frac{\Sigma |d|}{n}$$

$$= \frac{|d_1| + |d_2| + |d_3| + |d_4| + |d_5| + |d_6| + |d_7| + |d_8| + |d_9| + |d_{10}|}{10}$$

$$= \frac{0.4 + 0.4 + 0.3 + 0.2 + 0.1 + 0.1 + 0.2}{10}$$

$$= \frac{1.4}{10}$$

$$= \boxed{0.14\,\Omega}$$

5.) Probable Error :-

$= 0.67456 \times$ Standard.

$$r = 0.67456 \times 0.2\,\Omega$$

$$\boxed{r = 0.1349\,\Omega}$$

6.) Probable error of arithmetic error :-

$$r_m = \frac{r}{\sqrt{n-1}} = \frac{0.1349}{\sqrt{9}}$$

$$\boxed{r_m = 0.044\,\Omega}$$

7.) Probable error of Range :-

Range = Largest Value − Lowest Value.

Range = 517.51

$$\boxed{\text{Range} = 0.7}$$

Relative limitting error :-

$$\boxed{\delta A = A_a - A_s}$$

$$E_r = \frac{A_a - A_s}{A_s} = \frac{\text{Actual value} - \text{Nominal value}}{\text{Nominal value}}$$

$$\boxed{A_a = A_s \pm \delta A}$$

Q.3 The value of a Capacitance of a capacitor is 0.1 uF ± 5% manufacture. Find the limit between which the value of the capacitance gauranted.

Soln :- Here $A_s = 0.1\ \mu F$

$\% E_r = 5\%$

$E_r = 0.05$

$A_a = A_s (1 \pm E_r)$

$= 0.1\ (1 \pm 0.5)\ \mu F$

$= (0.1 \pm 0.005)\ \mu F$

$$\boxed{A_a = 0.95 \text{ to } 0.105\ \mu F}$$

Sources of Error :-

1.) Sampling error

2.) Instrument error.

3.) Systematic error.

4.) Arithmetic error.

Q.4 Given,

$x_1 = 21.7$ $x_6 = 21.9$

$x_2 = 22.0$ $x_7 = 22.0$

$x_3 = 21.8$ $x_8 = 21.9$

$x_4 = 22.0$ $x_9 = 22.5$

$x_5 = 22.1$ $x_{10} = 21.5$ volts.

Find 1.) The Mean
2.) Standard deviation
3.) Probable error of one reading
4.) Range
5.) Variance

1.) Mean :-

$$= \frac{21.7 + 22.0 + 21.8 + 22.0 + 22.1 + 21.9 + 22.0 + 21.9 + 22.5 + 21.5}{10}$$

$$= \frac{219.4}{10}$$

$$\boxed{\bar{x} = 21.94}$$

2.) Standard Deviation :-

* Deviation of each value :-

$d_1 = x_1 - \bar{x} = 21.7 = 21.94 = -0.24$

$d_2 = x_2 - \bar{x} = 22.0 - 21.94 = 0.06$

$d_3 = x_3 - \bar{x} = 21.8 - 21.94 = -0.14$

$d_4 = x_4 - \bar{x} = 22.0 - 21.94 = 0.06$

$d_5 = x_5 - \bar{x} = 22.1 - 21.94 = 0.16$

$d_6 = x_6 - \bar{x} = 21.9 - 21.94 = 0.04$

$d_7 = x_7 - \bar{x} = 22.0 - 21.94 = 0.06$

$d_8 = x_8 - \bar{x} = 21.9 - 21.94 = 0.04$

$d_9 = x_9 - \bar{x} = 22.5 - 21.94 = 0.56$

$d_{10} = x_{10} - \bar{x} = 21.5 - 21.94 = 0.44$

$$\sigma = \sqrt{\frac{d_1^2 + d_2^2 + d_3^2 + d_4^2 + d_5^2 + d_6^2 + d_7^2 + d_8^2 + d_9^2 + d_{10}^2}{10-1}}$$

$$\sigma = \sqrt{\frac{(-0.24)^2 + (0.06)^2 + (-0.14)^2 + (0.06)^2 + (0.16)^2 + (0.04)^2 + (0.04)^2 + (0.56)^2 + (0.44)^2}{9}}$$

$$\sigma = \sqrt{\frac{0.624}{9}}$$

$$\boxed{\sigma^2 = 0.0693}$$

3) Probable error of one reading :-

0.14839

$= \boxed{0.15 \text{ volts}}$

4) Range :-

largest value - lowest value

$= 22.5 - 21.5$

$= \boxed{1 \text{ Volt}}$

5.) Variance :-

$$(\sigma)^2 = 0.0729$$

$$r_m = \frac{r}{\sqrt{n-1}} = \boxed{\frac{0.15}{\sqrt{9}}}$$

▷ Calibration...

Instrument calibration is one of the primary processes used to maintain instrument accuracy.

Calibration is a comparison between a known measurement (the standard) and the measurement using instrument.

Purpose of Calibration:-

Calibration refers to the act of evaluating and adjusting the precision and accuracy of measurement equipment. Instrument calibration intended to eliminate or reduce bias in an instrument's reading over a range for all continous values.

Benefits of Calibration:-

~~Calibration refers to the act of~~

→ It determines whether measurement made before the calibration were valid.

→ It gives confidence that the future measurement will be accurate.

→ It assures consistency and compatibility with those made elsewhere.

→ Without calibration, the product quality may be poor thus opening up legal challanges and high failure rates of the products, thus increasing costs.

→ It increases efficiency by ensuring that measurement are correct.

Instrument
- Mechanical Instrument
- Electrical Instrument
  - Absolute Indirect
  - Secondary Direct
    - Analog Instrument
      - Deflecting Instrument
        - Indicating Instrument
        - Integrating Instrument
        - Recording Instrument
      - Null Deflection.
    - Digital Instrument
- Electronic Instrument

▶ Specification of Digital Multimeter :-

A digital multimeter (DMM) is a handheld device used to measure electrical parameters like voltage, current, resistance and more. It features a digital display, offers high accuracy and includes functions like auto-ranging, continuity testing and safety ratings (CAT I-IV). Key specs include measurement range, resolution, accuracy, True RMS capability and overload protection. It's versatile for troubleshooting and testing in electronics, electrical system, and industrial applications.