NETWORK THEOREMS

1. Superposition Theorem

Statement: In linear, active, bilateral network current flowing through any component is the algebraic sum of currents due to individual sources taking one at a time, replacing remaining sources with their internal impedances.

Proof: Consider the network shown in fig(1), in which Z_1, Z_2, Z_3 are impedances and V_1 and V_2 are applied sources.

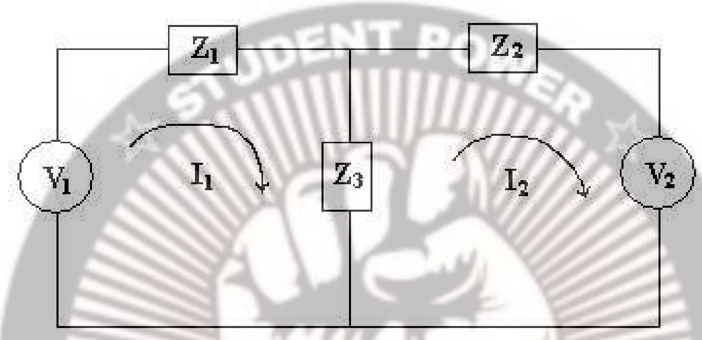


Fig. 1

Let I_1, I_2 be the mesh currents, writing mesh equations for fig(1).

$$(Z_1 + Z_3) I_1 - Z_3 I_2 = V_1 \quad \text{----------- (1)}$$

$$- Z_3 I_1 + (Z_2 + Z_3) I_2 = V_2 \quad \text{------------(2)}$$

Now keeping the source V_1 and replacing V_2 with its internal impedance. The modified ckt is shown in fig(2).

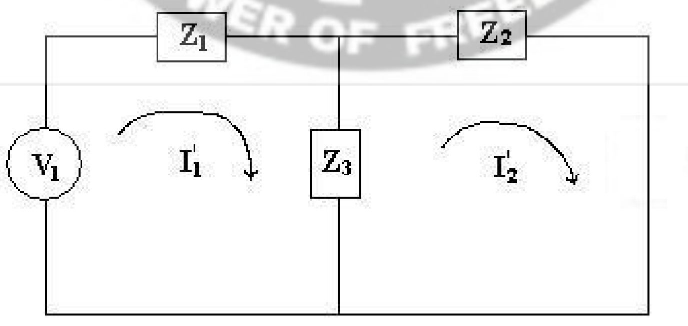


Fig. 2

Let I_1', I_2' be mesh currents, mesh equations for fig(2)

$$(Z_1 + Z_3)I_1' - Z_3 I_2' = V_1 \qquad \text{----------- (3)}$$

$$- Z_3 I_1' + (Z_2 + Z_3) I_2' = 0 \qquad \text{------------(4)}$$

Consider the V_2 and replace V_1 with its internal impedance as shown in fig(3)

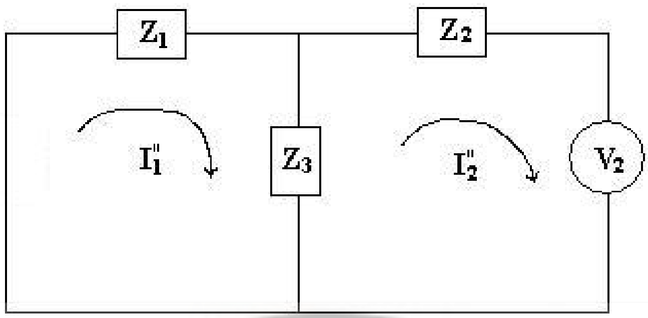


Fig.3

Let I_1'' , I_2'' be the mesh currents , mesh equations for fig(3),

$$(Z_1 + Z_3) I_1'' - Z_3 I_2'' = 0 \qquad \text{----------- (5)}$$

$$- Z_3 I_1'' + (Z_2 + Z_3) I_2'' = V_2 \qquad \text{------------(6)}$$

Adding eq(3) & eq(5) and eq(4) & eq(6), we get

$$(Z_1 + Z_3)(I_1' + I_2'') - Z_3 (I_2' + I_2'') = V_1 \qquad \text{-------------(7)}$$

$$- Z_3 (I_1' + I_1'') + (Z_2 + Z_3) (I_2' + I_2'') = V_2 \qquad \text{-------------(8)}$$

Comparing eq(1) & eq(7) and eq(2) & eq(8), we have

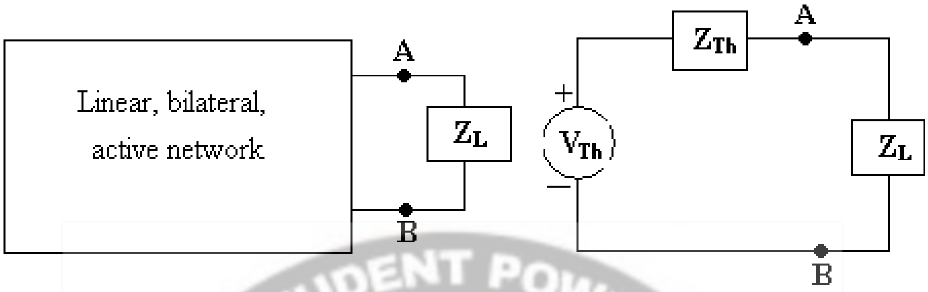
$$I_1 = (I_1' + I_2'')$$

$$I_2 = (I_2' + I_2'')$$

It is clear from the above equations that the current flowing through any component is algebraic sum of currents due to individual sources taking on at a time. Hence **Superposition theorem** is proved.

2. THEVENIN'S THEOREM

Statement: Any linear, bilateral, active network connected between two terminal A, B can be replaced by a voltage source supplying voltage V_{TH} in series with an impedance $Z_{TH.}$ Where V_{TH} known as Thevenin's voltage and is the open terminal voltage across A, B terminals. Z_{TH} is known as Thevenin's impedance and is measured across open terminals A, B by replacing all energy sources with their internal impedance.



Proof: Consider the network shown in fig(1), in which Z_1, Z_2 and Z_3 are impedances, V is the applied voltage and Z_L is the load impedance.

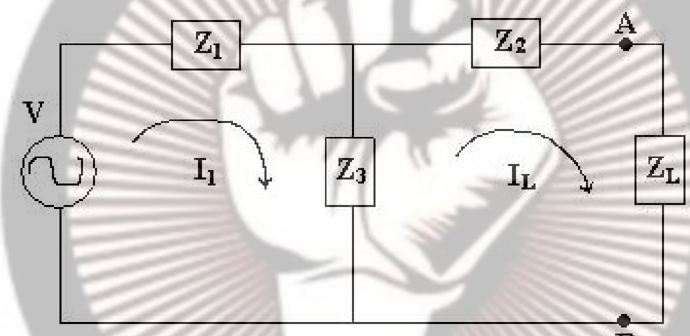


Fig. 1

Let I_1, I_L be the mesh currents, mesh equations for fig(1)

$$(Z_1 + Z_3) I_1 - Z_3 I_L = V \quad \text{----------------- (1)}$$

$$- Z_3 I_1 + (Z_2 + Z_3 + Z_L) I_L = 0 \quad \text{-----------------(2)}$$

From eq(2), we have

$$Z_3 I_1 = (Z_2 + Z_3 + Z_L) I_L$$

$$I_1 = \frac{(Z_2 + Z_3 + Z_L) I_L}{Z_3} \quad \text{——— (3)}$$

Substituting I_1 value from eq(3) in eq(1) we get

$$\frac{(Z_1+Z_3)(Z_2+Z_3+Z_L)I_L}{Z_3} - Z_3 I_L = V$$

$$\frac{(Z_1+Z_3)(Z_2+Z_3+Z_L)I_L - Z_3^2 I_L}{Z_3} = V$$

$$(Z_1+Z_3)(Z_2+Z_3+Z_L)I_L - Z_3^2 I_L = VZ_3$$

$$I_L\left[(Z_1+Z_3)(Z_2+Z_3+Z_L) - Z_3^2\right] = VZ_3$$

$$I_L = \frac{VZ_3}{\left[(Z_1+Z_3)(Z_2+Z_3+Z_L) - Z_3^2\right]} \qquad \text{———} \quad (4)$$

$$I_L = \frac{VZ_3}{\left[Z_1Z_2 + Z_1Z_3 + Z_1Z_L + Z_3Z_2 + Z_3^2 + Z_3Z_L - Z_3^2\right]}$$

$$I_L = \frac{VZ_3}{(Z_1Z_2 + Z_2Z_3 + Z_3Z_1) + Z_L(Z_1+Z_3)} \qquad \text{———} \quad (5)$$

Thevenin's Voltage (V_{Th})

To measure Thevenin's voltage make the terminals A, B open by removing the load impedance connected between them. The modified circuit diagram is shown in fig(2).

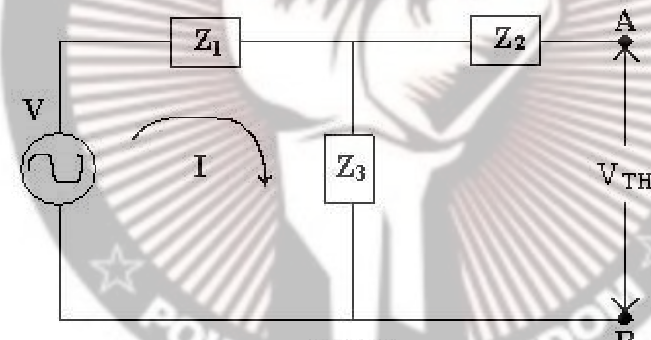


Fig. 2

voltage across terminals AB (V_{TH}) = voltage across Z_3

$$V_{TH} = I \times Z_3 \qquad \text{--------} \ (6)$$

But from fig(2)

$$I = \frac{V}{Z_1+Z_3}$$

Substituting I value in eq(6) we get

$$V_{TH} = \frac{VZ_3}{Z_1 + Z_3} \qquad \text{------} (7)$$

Thevenin's Impedance (Z_{TH})

Thevenin's impedance is measured across open terminals AB by replacing all energy sources with their internal impedances. The modified circuit diagram is shown in fig(3).

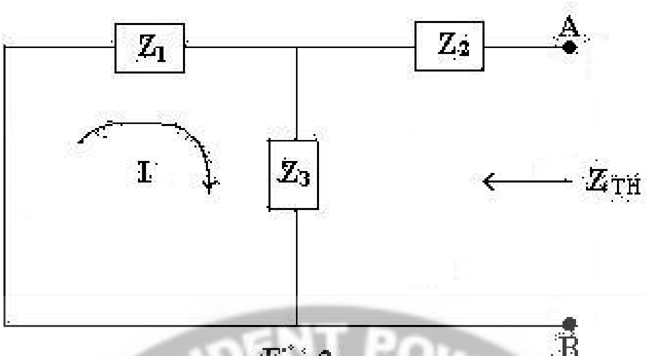


Fig. 3

$$Z_{TH} = (Z_1 \,||\, Z_3) + Z_2$$

$$Z_{TH} = \frac{Z_1 Z_3}{Z_1 + Z_3} + Z_2$$

$$Z_{TH} = \frac{Z_1 Z_3 + Z_2(Z_1 + Z_3)}{Z_1 + Z_3}$$

$$Z_{TH} = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_1 + Z_3} \qquad \text{------} (8)$$

Thevenin's Equivalent Circuit

Thevenin's equivalent circuit can be constructed by connecting voltage source supplying voltage V_{TH} in series with an impedance Z_{TH} as shown in fig.(4). Finally connect the load impedance which was removed from the circuit.

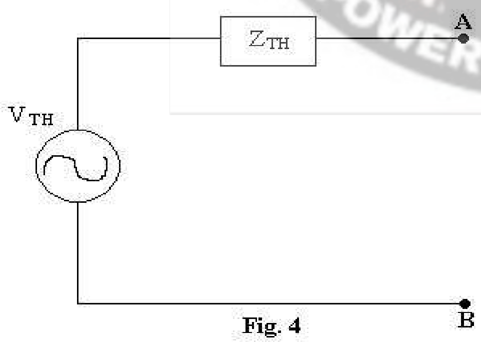


Fig. 4

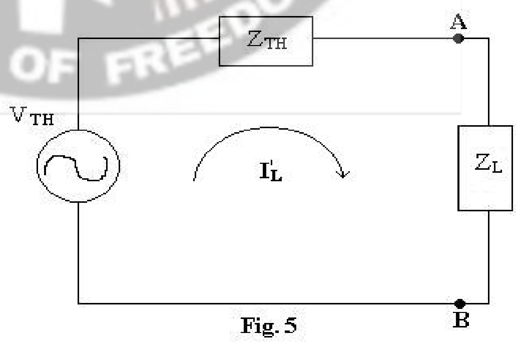


Fig. 5

The current in the Thevenin's equivalent circuit is given by

$$I' = \frac{V_{TH}}{Z_{TH} + Z_L} \qquad ------(9)$$

Substituting V_{TH} and Z_{TH} values in eq (9), we get

$$I_L' = \frac{\frac{VZ_3}{Z_1 + Z_3}}{\frac{(Z_1Z_2 + Z_2Z_3 + Z_3Z_1)}{(Z_1 + Z_3)} + Z_L}$$

$$I_L' = \frac{\frac{VZ_3}{Z_1 + Z_3}}{\frac{(Z_1Z_2 + Z_2Z_3 + Z_3Z_1) + Z_L(Z_1 + Z_3)}{(Z_1 + Z_3)}}$$

$$I_L' = \frac{VZ_3}{(Z_1Z_2 + Z_2Z_3 + Z_3Z_1) + Z_L(Z_1 + Z_3)} \qquad \text{———} \quad (10)$$

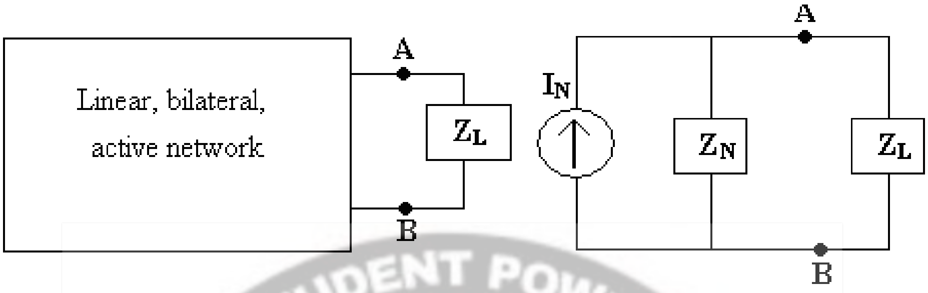
Comparing eq(5) & eq(10), we get

$$I_L = I_L'$$

That is the current flowing through load resistor in a given circuit is equal to the current flowing through the load resistor in Thevenin's equivalent circuit. Hence Thevenin's theorem is proved.

3. NORTON'S THEOREM

Statement: Any linear, bilateral, active network connected between two terminal A, B can be replaced by a current source supplying current I_N in parallel with an impedance Z_N. Where I_N is known as Norton's and is measured by short circuiting the terminals AB. Z_N is known as Norton's impedance and is measured across open terminals A, B by replacing all energy sources with their internal impedance.



Proof: Consider the network shown in fig(1), in which Z_1, Z_2 and Z_3 are impedances, V is the applied voltage and Z_L is the load impedance.

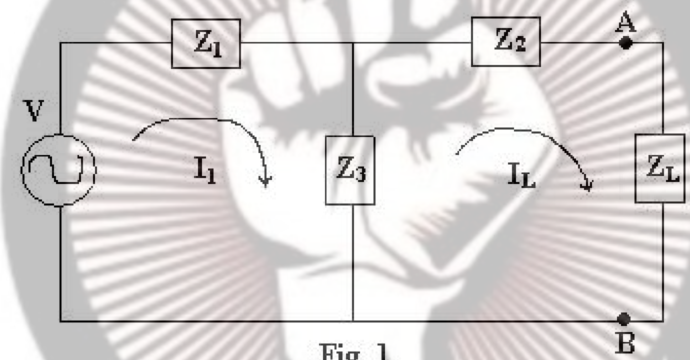


Fig. 1

Let I_1, I_L be the mesh currents, mesh equations for fig(1)

$$(Z_1 + Z_3) I_1 - Z_3 I_L = V \quad \text{------------------ (1)}$$

$$-Z_3 I_1 + (Z_2 + Z_3 + Z_L) I_L = 0 \quad \text{------------------(2)}$$

From eq(2), we have

$$Z_3 I_1 = (Z_2 + Z_3 + Z_L) I_L$$

$$I_1 = \frac{(Z_2 + Z_3 + Z_L)\, I_L}{Z_3} \quad \text{——— (3)}$$

Substituting I_1 value from eq(3) in eq(1) we get

$$\frac{(Z_1+Z_3)(Z_2+Z_3+Z_L)\,I_L}{Z_3} - Z_3\,I_L = V$$

$$\frac{(Z_1+Z_3)(Z_2+Z_3+Z_L)\,I_L - Z_3^2\,I_L}{Z_3} = V$$

$$(Z_1+Z_3)(Z_2+Z_3+Z_L)\,I_L - Z_3^2\,I_L = V Z_3$$

$$I_L\left[(Z_1+Z_3)(Z_2+Z_3+Z_L) - Z_3^2\right] = V Z_3$$

$$I_L = \frac{V Z_3}{\left[(Z_1+Z_3)(Z_2+Z_3+Z_L) - Z_3^2\right]} \qquad \text{———— } (4)$$

$$I_L = \frac{V Z_3}{\left[Z_1Z_2 + Z_1Z_3 + Z_1Z_L + Z_3Z_2 + Z_3^2 + Z_3Z_L - Z_3^2\right]}$$

$$I_L = \frac{V Z_3}{(Z_1Z_2 + Z_2Z_3 + Z_3Z_1) + Z_L(Z_1+Z_3)} \qquad \text{———— } (5)$$

Norton's Current (I_N)

To measure Norton's current short circuit the terminals A, B open by replacing the load impedance with a good conductor between them. The modified circuit diagram is shown in fig(2).

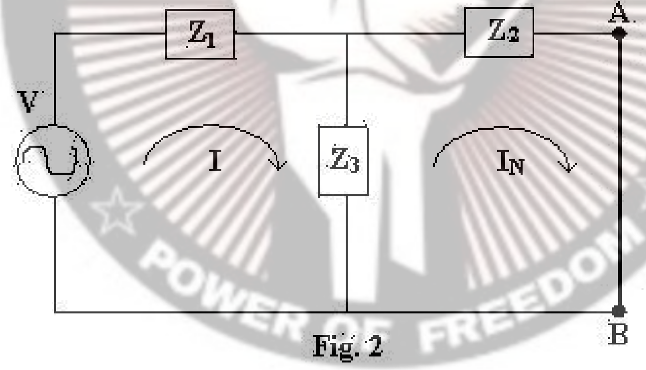


Fig. 2

Let I, I_N be the mesh currents, writing mesh equations for fig(2)

$(Z_1 + Z_3)\, I - Z_3 I_N = V$ ------------- (6)

$-Z_3 I + (Z_2 + Z_3) I_N = 0$ -------------(7)

From eq(7) we have

$-\quad Z_3 I = -(Z_2 + Z_3) I_N$

Or $Z_3 I = (Z_2 + Z_3) I_N$

$$I = \frac{(Z_2 + Z_3)I_N}{Z_3} \qquad -----(8)$$

Substituting I value from eq(8) in eq(6) we get

$$(Z_1 + Z_3)\,\frac{(Z_2 + Z_3)I_N}{Z_3} - Z_3 I_N = V$$

$$\frac{(Z_1 + Z_3)(Z_2 + Z_3)I_N - Z_3^2 I_N}{Z_3} = V$$

$$(Z_1 + Z_3)(Z_2 + Z_3)I_N - Z_3^2 I_N = VZ_3$$

$$I_N\left[(Z_1 + Z_3)(Z_2 + Z_3) - Z_3^2\right] = VZ_3$$

$$I_N = \frac{VZ_3}{\left[(Z_1 + Z_3)(Z_2 + Z_3) - Z_3^2\right]}$$

$$I_N = \frac{VZ_3}{\left[(Z_1Z_2 + Z_3Z_1 + Z_3Z_2 + Z_3^2 - Z_3^2\right]}$$

$$I_N = \frac{VZ_3}{\left[(Z_1Z_2 + Z_3Z_1 + Z_3Z_2)\right]} \qquad -----(9)$$

NORTON'S IMEDANCE (Z_N)

Norton's impedance is measured across open terminals A,B by replacing all energy sources with their internal impedances. The modified circuit diagram is shown in fig(3).

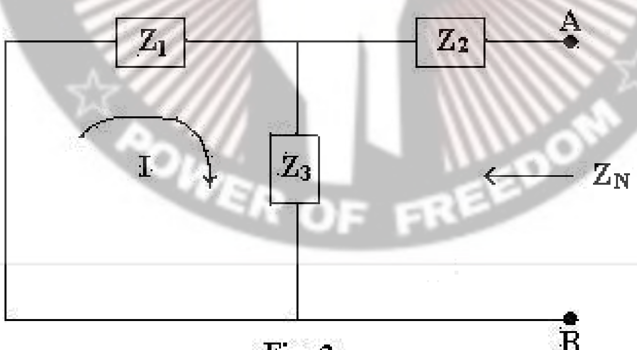


Fig. 3

$Z_N = (Z_1 \parallel Z_3) + Z_2$

$$Z_N = \frac{Z_1 Z_3}{(Z_1 + Z_3)} + Z_2$$

$$Z_N = \frac{Z_1 Z_3 + Z_2(Z_1 + Z_3)}{(Z_1 + Z_3)}$$

$$Z_N = \frac{(Z_1 Z_2 + Z_3 Z_1 + Z_3 Z_2)}{(Z_1 + Z_3)}$$

Norton's Equivalent Circuit

Norton's equivalent circuit can be constructed by connecting current source supplying current I_N parallel with an impedance Z_N as shown in fig.(4). Finally connect the load impedance which was removed from the circuit.

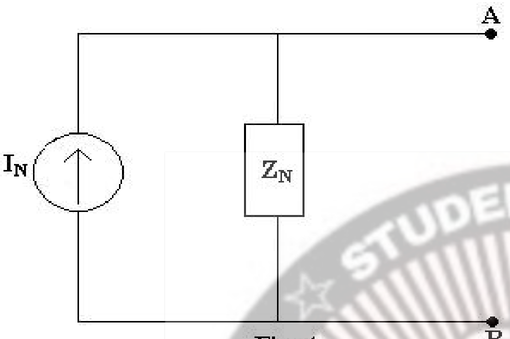


Fig. 4

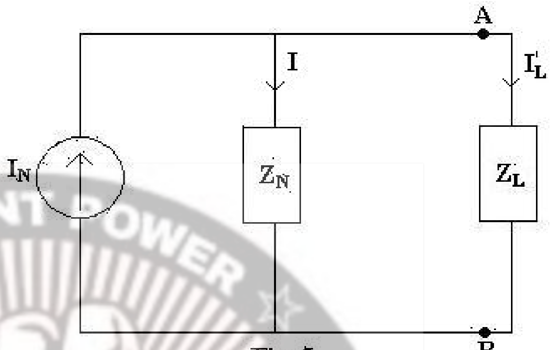


Fig. 5

The current in the Norton's equivalent circuit is given by

$$I_L' = \frac{Z_N}{Z_N + Z_L} . I_N \qquad ------(9)$$

Substituting I_N and Z_N values in eq (9), we get

$$I_L' = \frac{\frac{(Z_1 Z_2 + Z_3 Z_1 + Z_3 Z_2)}{(Z_1 + Z_3)}}{\frac{(Z_1 Z_2 + Z_3 Z_1 + Z_3 Z_2)}{(Z_1 + Z_3)} + Z_L} . \frac{VZ_3}{[(Z_1 Z_2 + Z_3 Z_1 + Z_3 Z_2]}$$

$$I_L' = \frac{\frac{(Z_1 Z_2 + Z_3 Z_1 + Z_3 Z_2)}{(Z_1 + Z_3)}}{\frac{(Z_1 Z_2 + Z_3 Z_1 + Z_3 Z_2) + Z_L(Z_1 + Z_3)}{(Z_1 + Z_3)}} . \frac{VZ_3}{[(Z_1 Z_2 + Z_3 Z_1 + Z_3 Z_2]}$$

$$I_L' = \frac{(Z_1 Z_2 + Z_3 Z_1 + Z_3 Z_2)}{(Z_1 + Z_3)} . \frac{(Z_1 + Z_3)}{[(Z_1 Z_2 + Z_3 Z_1 + Z_3 Z_2) + Z_L(Z_1 + Z_3)]} . \frac{VZ_3}{[(Z_1 Z_2 + Z_3 Z_1 + Z_3 Z_2]}$$

$$I'_L = \frac{VZ_3}{(Z_1Z_2 + Z_2Z_3 + Z_3Z_1) + Z_L(Z_1 + Z_3)} \quad \text{———— (10)}$$

Comparing eq(5) & eq(10), we get

$$I_L = I'_L$$

That is the current flowing through load resistor in a given circuit is equal to the current flowing through the load resistor in Norton's equivalent circuit. Hence Norton's theorem is proved.

4. Maximum Power Transfer Theorem (DC)

Statement: In linear, bilateral, active network power delivered to a load connected between two terminals is maximum when the load resistance is equal to internal resistance of the source.

Proof: Consider circuit shown in fig(1) in which R_i, R_L are internal resistance, load resistance respectively and V is the applied voltage.

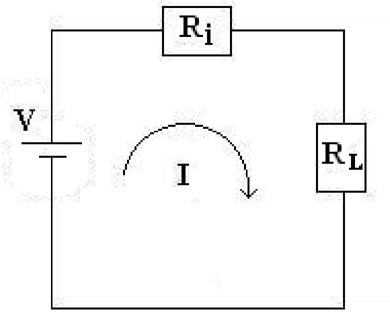


Fig. 1

Let I is the current in the circuit given by

$$I = \frac{V}{R_i + R_L} \quad \text{------ (1)}$$

Power delivered to the load is given by $P = I^2 R_L$ ------- (2)

Substituting I value from Eq(1) in Eq(2) we get

$$P = \left(\frac{V}{R_i + R_L}\right)^2 R_L$$

$$P = \frac{V^2 R_L}{(R_i + R_L)^2} \quad \text{------- (3)}$$

The variation of the power with the load resistance is $\frac{dp}{dR_L}$

$$\frac{dp}{dR_L} = \frac{d}{dR_L}\left(\frac{V^2 R_L}{(R_i + R_L)^2}\right)$$

$$\frac{dp}{dR_L} = \frac{(R_i + R_L)^2 V^2 - V^2 R_L 2(R_i + R_L)}{(R_i + R_L)^4} \quad \text{------- (4)}$$

Condition for maximum power transfer is $\frac{dp}{dR_L} = 0$

$$\frac{(R_i + R_L)^2 V^2 - V^2 R_L 2(R_i + R_L)}{(R_i + R_L)^4} = 0$$

$$(R_i + R_L)V^2 \left[(R_i + R_L) - 2R_L\right] = 0$$

$$R_i + R_L - 2R_L = 0$$

$$R_i - R_L = 0$$

$$R_L = R_i \quad \text{------ (5)}$$

From Eq(5), it is clear that the load resistance is equal to the internal resistance of the source, hence maximum power transfer theorem is proved.

5. Maximum Power Transfer Theorem (AC)

Statement: In an active network power delivered to a load is maximum if load impedance is complex conjugate of internal impedance of the source.

Proof: Let us consider the network shown in fig(1), in which V is the applied voltage, Z_i, Z_L are internal impedance, load impedance respectively. Z_i and Z_L are given by

$$Z_i = R_i + jX_i \quad \text{---------} \ (1)$$

$$Z_i = R_L + jX_L \ \text{---------} \ (2)$$

where X_i, X_L are reactances.

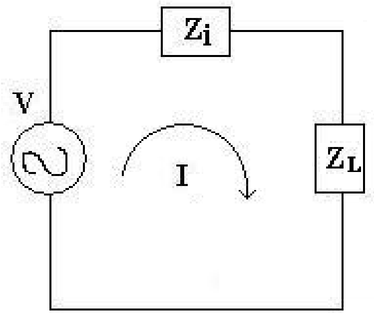


Fig. 1

Current in the circuit is given by

$$I = \frac{V}{Z_i + Z_L} \quad \text{------} \ (1)$$

Substituting Z_i, Z_L values in Eq(1) we get

$$I = \frac{V}{(R_i + jX_i) + (R_L + jX_L)}$$

$$\text{Or} \quad I = \frac{V}{(R_i + R_L) + j(X_i + X_L)} \quad \text{------} \ (2)$$

Taking modulus on both sides of Eq(2), we get

$$|I|^2 = \sqrt{\frac{V^2}{(R_i + R_L)^2 + (X_i + X_L)^2}}$$

$$\text{Or} \quad |I|^2 = \frac{V \ R_L}{\sqrt{\left[(R_i + R_L)^2 + (X_i + X_L)^2\right]}} \quad \text{------} \ (3)$$

Power delivered to the load is $P = |I|^2 R_L$ ---------- (4)

Substituting Eq(3) in Eq(4) we get

$$P=\frac{V^2R_L}{\left[(R_i+R_L)^2+(X_i+X_L)^2\right]} \text{ --------- } (5)$$

Power variation with X_L is $\frac{dP}{dX_L}$ and is given by

$$\frac{dP}{dX_L}=\frac{d}{dX_L}\left[\frac{V^2R_L}{\left[(R_i+R_L)^2+(X_i+X_L)^2\right]}\right]$$

$$\frac{dP}{dX_L}=\frac{V^2R_L\,2(X_i+X_L)}{\left[(R_i+R_L)^2+(X_i+X_L)^2\right]^2}$$

The condition for maximum power transfer is $\frac{dP}{dX_L}=0$

$$\therefore\frac{V^2R_L\,2(X_i+X_L)}{\left[(R_i+R_L)^2+(X_i+X_L)^2\right]^2}=0$$

$$\text{or}\quad V^2R_L\,2(X_i+X_L)=0$$

$$(X_i+X_L)=0$$

$$\text{or}\quad X_i=-X_L \text{ -------- } (6)$$

Power variation with R_L is $\frac{dP}{dR_L}$

$$\frac{dP}{dR_L}=\frac{d}{dR_L}\left[\frac{V^2R_L}{\left[(R_i+R_L)^2+(X_i+X_L)^2\right]}\right]$$

$$\frac{dP}{dR_L}=\frac{\left[(R_i+R_L)^2+(X_i+X_L)^2\right]V^2-V^2R_L\,2(R_i+R_L)}{\left[(R_i+R_L)^2+(X_i+X_L)^2\right]}$$

The condition for maximum power transfer is $\frac{dP}{dR_L}=0$

$$\frac{\left[(R_i+R_L)^2+(X_i+X_L)^2\right]V^2-V^2R_L\,2(R_i+R_L)}{\left[(R_i+R_L)^2+(X_i+X_L)^2\right]}=0$$

$$\left[(R_i+R_L)^2+(X_i+X_L)^2\right]V^2-V^2R_L\,2(R_i+R_L)=0 \text{ ------ } (7)$$

Substituting $X_i = -X_L$ in Eq(7) we get

$$\left[(R_i + R_L)^2 + (-X_L + X_L)^2\right] V^2 - V^2 R_L 2(R_i + R_L) = 0$$

$$\text{or} \qquad (R_i + R_L)^2 V^2 - V^2 R_L 2(R_i + R_L) = 0$$

$$\text{or} \quad (R_i + R_L) - 2R_L = 0$$

$$\text{or} \quad R_i - R_L = 0$$

$$\text{or} \quad R_i = R_L \quad \text{---------} \ (8)$$

substituting X_i and R_i values in Eq(1), we get

$$Z_i = R_L - jR_L \quad \text{---------} \ (9)$$

It is clear that the Eq(2) and Eq(7) are complex conjugates. The load impedance is complex conjugate of the load impedance of the source, hence maximum power transfer theorem is proved.

Maximum power delivered to the load can be obtained by substituting $X_i = -X_L$ in Eq(5), we get

$$P_{Max} = \frac{V^2 R_L}{\left[(R_i + R_L)^2 + (-X_L + X_L)^2\right]}$$

$$\text{or} \qquad P_{Max} = \frac{V^2 R_L}{\left[(R_i + R_L)^2\right]}$$

substituting $R_i = R_L$ in the above equation, we get

$$P_{Max} = \frac{V^2 R_L}{\left[(R_L + R_L)^2\right]}$$

$$\text{or} \qquad P_{Max} = \frac{V^2 R_L}{(2R_L)^2}$$

$$\text{or} \qquad P_{Max} = \frac{V^2 R_L}{4{R_L}^2}$$

$$\text{or} \qquad \boxed{P_{Max} = \frac{V^2{}_L}{4R_L}}$$

6. Reciprocity Theorem

Statement: In any linear, bilateral, active network voltage applied in one mesh and the resulting current in other mesh are interchangeable.

Proof: Consider a network shown in fig(1), in which voltage (V) is applied in the first mesh. The mesh currents are I_1 and I_2. Now the voltage source is shifted into the second mesh as shown in fig(2). The mesh currents are $I_1^{'}$ and $I_2^{'}$.

According to reciprocity theorem $I_2 = I_1^{'}$

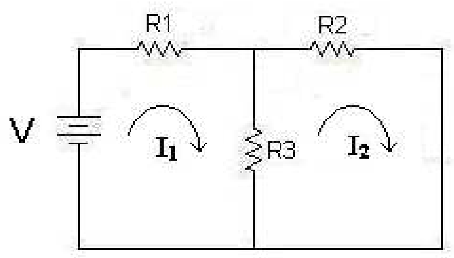
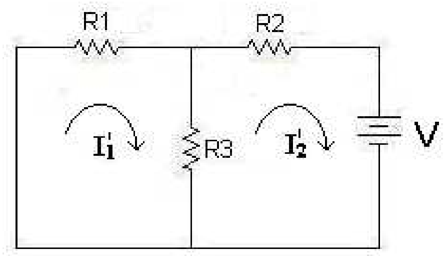


fig (1)



fig(2)

Mesh equations for fig(1)

$$(R_1+R_3)I_1 - R_3I_2 = V \text{ ---------- (1)}$$

$$-R_3I_1 + (R_2+R_3)I_2 = 0 \text{ ---------- (2)}$$

From Eq (2), we have

$$-R_3I_1 = -(R_2+R_3)I_2$$

$$\text{or} \quad R_3I_1 = (R_2+R_3)I_2$$

$$\text{or} \quad I_1 = \frac{(R_2+R_3)I_2}{R_3} \text{ ----- (3)}$$

substituting I_1 value from Eq(3) in Eq(1), we get

$$(R_1+R_3)\frac{(R_2+R_3)I_2}{R_3} - R_3I_2 = V$$

$$\text{or} \quad \frac{(R_1+R_3)(R_2+R_3)I_2 - R_3^2I_2}{R_3} = V$$

$$\text{or} \quad \frac{(R_1R_2+R_3R_2+R_3R_1+R_3^2)I_2 - R_3^2I_2}{R_3} = V$$

$$\text{or} \quad I_2\left[R_1R_2+R_3R_2+R_3R_1+R_3^2-R_3^2\right]=VR_3$$

$$\text{or} \quad I_2\left[R_1R_2+R_3R_2+R_3R_1\right]=VR_3$$

$$\text{or} \quad I_2=\frac{VR_3}{\left(R_1R_2+R_3R_2+R_3R_1\right)} \text{ ------ (4)}$$

Mesh equations for fig(2)

$$(R_1+R_3)I_1^{'}-R_3I_2^{'}=0 \text{ ------- (5)}$$

$$-R_3I_1^{'}+(R_2+R_3)I_2^{'}=V \text{ ------ (6)}$$

From Eq(5), we have

$$R_3I_2^{'}=(R_1+R_3)I_1^{'}$$

$$\text{or} \quad I_2^{'}=\frac{(R_1+R_3)I_1^{'}}{R_3} \text{ ------ (7)}$$

Substituting $I_2^{'}$ value from Eq(7) in Eq(6), we get

$$-R_3I_1^{'}+(R_2+R_3)\frac{(R_2+R_3)I_1^{'}}{R_3}=V$$

$$\text{or} \quad \frac{-R_3^2I_1^{'}+(R_2+R_3)(R_1+R_3)I_1^{'}}{R_3}=V$$

$$\text{or} \quad \frac{-R_3^2I_1^{'}+(R_1R_2+R_2R_3+R_3R_1+R_3^2)I_1^{'}}{R_3}=V$$

$$\text{or} \quad I_1^{'}\left[-R_3^2+R_1R_2+R_2R_3+R_3R_1+R_3^2\right]=VR_3$$

$$\text{or} \quad I_1^{'}\left[R_1R_2+R_2R_3+R_3R_1\right]=VR_3$$

$$\text{or} \quad I_1^{'}=\frac{VR_3}{\left(R_1R_2+R_2R_3+R_3R_1\right)} \text{ ----- (8)}$$

comparing Eq(4) and Eq(8), we get

$$I_2=I_1^{'}$$

It is clear from the above equation that the applied voltage in one mesh and the resulting current in other mesh are interchangeable, hence **reciprocity theorem** is proved.