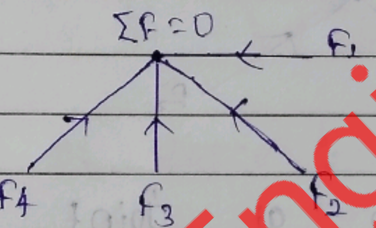
*. Force :-

Force is a push and pull apply over at object.

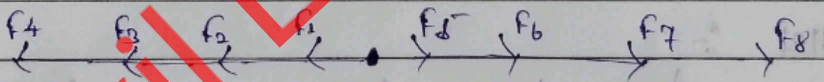
*. System of force / force system :-

When a number force more than two is apply over an object thats called force system.

Eg :- (i)

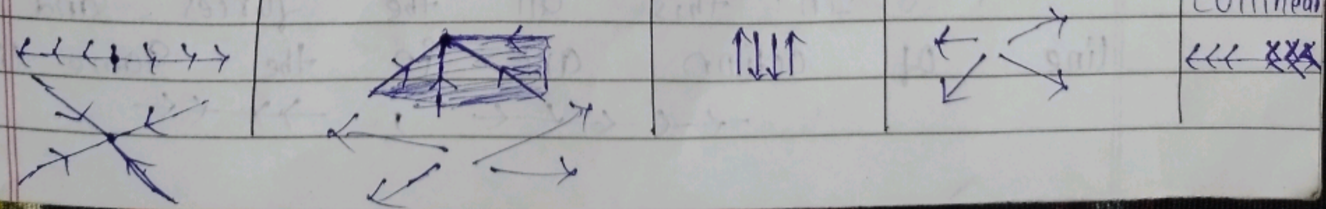


(ii)



*. Classification of force system :-

- Coplaner
  - Concurrent
  - Non-Concurrent
  - Parallel
  - Non-Parallel
  - Collinear
- Non-Coplaner



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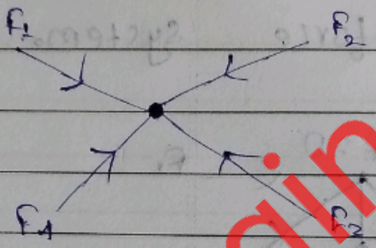
*. Coplaner :-

The system of force in which all force are apply on same plane

↳ Concurrent :-

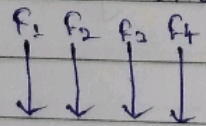
All the force are coplaner and line of action of all force intersect at a point.

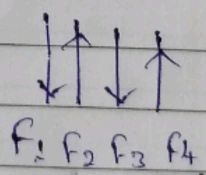


eg :- Force system at joint of truss

↳ Parallel :-

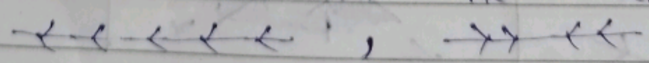
All the force at their line of action are parallel to each other either in same all opposite direction.





↳ Collinear :-

In this all the forces and their line of action are in the same line



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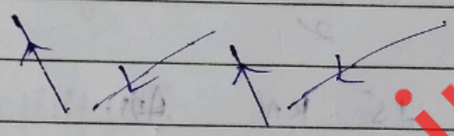
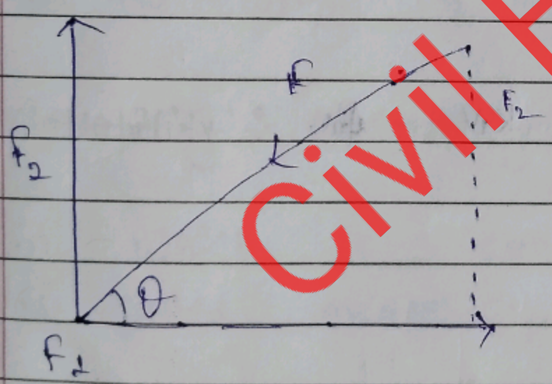
Eg :- force system on rope in tug of war game.

↳ Non-concurrent / non-parallel :- In this all forces are act on same plane but diffent in direction and cannot intersect at a singal point.



*. Resolution of force :-



$$\sin\theta = \frac{F_2}{F} \Rightarrow F_2 = F\sin\theta$$

$$\cos\theta = \frac{F_1}{F} \Rightarrow F_1 = F\cos\theta$$

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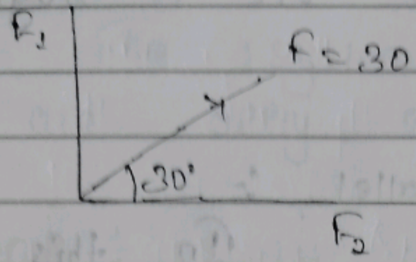
Ex :-

(i)



Sol^n

$F_1 = F \sin 30°$

$= 30 \times \frac{1}{2}$

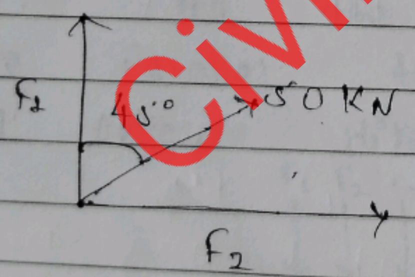
$= 15$ KN Ans.

$F_2 = F \cos 30°$

$= 30 \times \frac{\sqrt{3}}{2}$

$= 15\sqrt{3}$ KN Ans.

(ii)



Sol^n

$F_1 = F \cos 45°$

$= 50 \times \frac{1}{\sqrt{2}}$

~~= 25 KN Ans.~~ $= \sqrt{2} \times \sqrt{2} \times 25 \times \frac{1}{\sqrt{2}}$

$= 25\sqrt{2}$ KN Ans.

$$F_2 = F \sin 45^\circ$$

$$= 50 \times \frac{1}{\sqrt{2}}$$

$$= \sqrt{2} \times \sqrt{2} \times 25 \times \frac{1}{\sqrt{2}}$$

$$= 25\sqrt{2} \text{ kN} \quad \text{Ans}$$

SOM

* Properties of Material :

↳ Ductile / Ductility :

It is the properties of material in which a material can drawn into thin wire to the apply a load. EX :- Iron rod, bitumen or tar

↳ ~~Mellability~~ : Malleability :-

It is the properties of material in which it can deform into a thin sheet is called malleability. EX :- Alumminum, copper etc

↳ Brittle / Rigid :-

It is properties of material in which cannot be deform upto to failure or break ~~apply~~ after apply load

ex :- Glass, tiles, bricks etc.

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↳ Hardness :-

It is the property of material in which check the strength against mutual rubbing over the material after applying Load and the test is abrasion (Los Angele's)

↳ Impact :-

In this which check the strength of material to resist the sudden load (impact load) applyied over them.

↳ ~~Elascity~~ Elasticity :-

It is the property of material in which a body can deform after apply the load and then regain its ~~orgi~~ original shape and and size after removed of loads.

↳ Plasticity :-

It is property of material in which a body can deform after apply load and then it does not regain its original ~~sp~~ shape and size after removed of loads.

↳ **Fatigue** :
It is Property of material which make its counter the Continuous loading overthem

↳ **Creep** :
It is the constant load over the material during its intire life span it consider as the deffect on dead load.

↳ **Compressibility** : (Ability to resist compression).
In this material is enough strong to resist the compression or compressive load overthem.

✱. **Stress** :

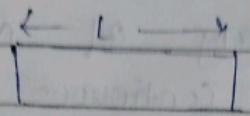
$$\sigma = \frac{\text{Load}}{\text{Area}}$$

$$\sigma = \frac{P}{A} \text{ or } \frac{F}{A}$$

S.I unit = N/mm^2

↳ It is define as the internal power which develop to contour or resist the external load and Stress is the ratio of load & Area.

*. Strain :

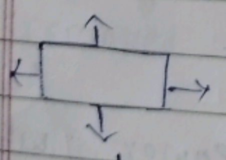


*. Types of Stress :

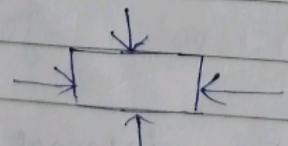
→ There are two types of stress

- **Normal stress (⊥)**
  - Tensile
  - compressive
- **Shear stress (∥)**

Tensile

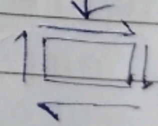


compressive



$\sigma = \frac{P}{A}$ N/mm²

Shear stress (∥)



$\tau = \frac{P}{A} = \frac{\text{Shear load}}{\text{Area}}$

N/mm²

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↳ Normal stress :-

Normal stress is develop due to normal force either tensile or compressive which is perpendicular (⊥) to the surface.

↳ Shear stress :-

It is develop due to shear force which parallel (∥) to the surface.

*. ~~Ho~~ Hooke's law :

→ It state that stress is directly proportional to the strain upto the limit of proporonality

$$\Rightarrow \sigma \propto \epsilon$$

$$\Rightarrow \sigma = C\epsilon$$

$$\boxed{C = \frac{\sigma}{\epsilon}}$$

$C \Rightarrow$ Elastic constant

a) E = young's modulus or Modulus of elesticity

$$E = \frac{\text{Normal stress}}{\text{Normal strain}} = \frac{\sigma}{\epsilon}$$

*. Type of strain :

There are two types of strain

Normal strain | Shear strain

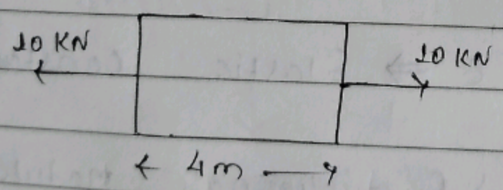
↳ **Normal strain** :- strain which develope due to normal force is called normal strain.

(i) Tensile strain :- strain occur due to tensile force.

(ii) Compressive strain :- strain occur due to compressive force.

↳ Shear strain :- Strain occur due to shear force is called shear strain.

Q.



Box of size = 3×5 m

Final length = 4.3 m

sol^n

$$\sigma_T = \frac{P}{A} = \frac{10\ KN}{3\times5} = \frac{10}{15} = 0.067\ KN/m^2$$

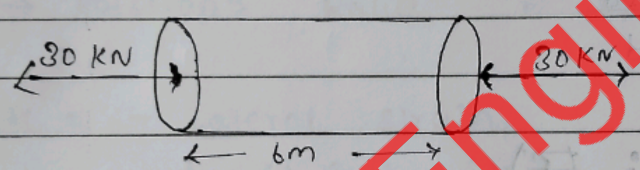
$\epsilon = \frac{\Delta L}{L}$ or $\frac{\text{final length} - \text{Intial Length}}{\text{Intial length}}$

$= \frac{4.3 - 4}{4}$

$= \frac{0.3}{4}$

$= 0.075$

Q.



Diameter = 5m

final length = 6.5 m

Sol^n

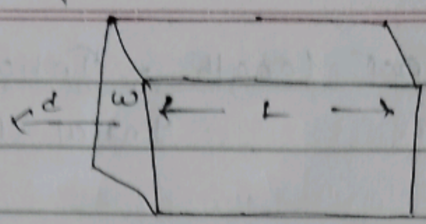
$\sigma = \frac{P}{A} = \frac{30\text{ KN}}{\pi/4 \times D^2} = \frac{30}{\pi/4 \times 5^2}$

$= \frac{\cancel{30}^{6} \times 4}{\pi \times \cancel{25}_{5}} = \frac{24 \times \cancel{100}^{20}}{3.14 \times \cancel{5}} = \frac{480}{3.14}$

$= 1.528$ KN/m^2 Ans.

$\epsilon = \frac{6.5 - 6}{6} = \frac{0.5}{6}$

$= 0.083$ Ans.



$$\epsilon_L = \frac{+\Delta L}{L}$$

$$\epsilon_w = \frac{-\Delta w}{w}$$

$$\epsilon_d = \frac{-\Delta d}{d}$$

*. Elastic constant : (E)

a) Modulus of ~~rigidity~~ elasticity or young's modulus

$$E = \frac{\text{Normal stress}}{\text{Normal strain}} \quad (T \text{ or } C)$$

b) Modulus of rigidity (G) = $\frac{\text{Shear stress}}{\text{Shear strain}}$

c) Modulus of Bulk (K) = $\frac{\text{Bulk stress}}{\text{Bulk strain}}$

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Relation between E, K, G & μ

a.) $E = 3K(1-2\mu)$

b.) $E = 2G(1+\mu)$

c.)

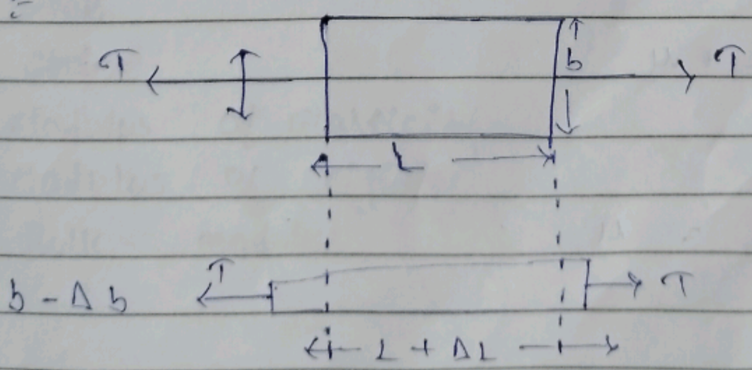
- $\mu \rightarrow$ Poission's Ratio

$$\mu = -\frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

- **Poission's effect :**

On applying a force on one side of an object or ~~ob~~ in one direction the respected dimension get change (increase or decrease), too make the volume constant the other side dimension get inversely affected

Eg :-



, length get increased but width get decrease.

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→ Relation b/w E, K & G :

$$E = 3K(1 - 2\mu)$$

$$\Rightarrow \frac{E}{3K} = (1 - 2\mu)$$

$$\Rightarrow \frac{E}{3K} - 1 = -2\mu$$

$$\Rightarrow 1 - \frac{E}{3K} = 2\mu$$

$$\Rightarrow \frac{3K - E}{3K} = 2\mu$$

$$\Rightarrow \frac{3K - E}{6K} = \mu \text{ ——— eq (i)}$$

$$\boxed{E = 2G(1 + \mu)}$$

$$\Rightarrow \frac{E}{2G} = 1 + \mu$$

$$\Rightarrow \frac{E}{2G} - 1 = \mu$$

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$$\Rightarrow \frac{E - 2G}{2G} = \mu \quad \text{--- (ii)}$$

From eqn (i) & (ii)

$$\Rightarrow \frac{3K - E}{6K} = \frac{E - 2G}{2G}$$

$$\Rightarrow 2G(3K - E) = 6K(E - 2G)$$

$$\Rightarrow 6KG - 2EG = 6KE - 12KG$$

$$\Rightarrow 18KG = E(2G + 6K)$$

$$\Rightarrow \overset{9}{\cancel{18}}KG = \cancel{2}E(G + 3K)$$

$$\therefore \boxed{E = \frac{9KG}{3K + G}}$$

Q. Determine : ~~the~~ S

a.) Stress

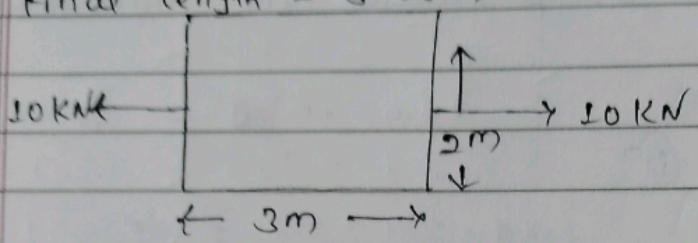
b.) Strain

c.) Modulus of elasticity

d.) Modulus of rigidity

e.) Bulk modulus

9.) Final length = 3.05 m



Take $\mu = 0.5$

sol^n

a.) Stress $= \dfrac{\text{load}}{\text{Area}}$

$= \dfrac{10 \text{ KN}}{3 \times 2} = \dfrac{10}{6} \text{ KN/m}^2$

$= 1.67 \text{ KN/m}^2$

b.) strain $= \dfrac{F.L - I.L}{I.L}$

$= \dfrac{3.05 - 3}{3}$

$= \dfrac{0.05}{3}$

$= 0.0167$

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(c) Modulus of elasticity $= \frac{\text{Normal stress}}{\text{Normal strain}} = \frac{\cancel{1.667} \times 100}{0.01\cancel{667}}$

$= 100 \ KN/m^2$

(d) $G = \frac{\text{Shear stress}}{\text{Shear strain}} \Rightarrow E = 2G(1 + \mu)$

$\Rightarrow 100 = 2G(1 + 0.5)$

$\Rightarrow 50 = 1.5 \ G$

$\Rightarrow G = \frac{50}{1.5}$

$= 33.33 \ KN/m^2$ Ans.

(e) ~~$K = 3K(1 - 2\mu)$~~

~~$100 = 3K(1 - 2 \times 0.5)$~~

~~$100 = 3K(1 - 1.0)$~~

~~$100 = 3K \times 0$~~

~~$K = 100 \ KN/m^2$ Ans.~~

~~$K = \frac{100}{3}$~~

~~$= 33.33 \ KN/m^2$ Ans.~~

$K = 1.$

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Q) $K = ?$

$$E = 3K(1-2\mu)$$

$$100 = 3K(1 - 2 \times 0.5)$$

$$100 = 3K(1-1)$$

$$100 = 3K(0)$$

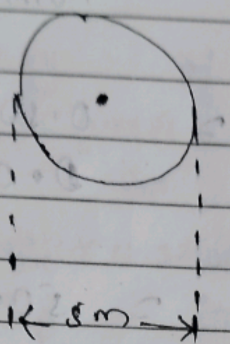
$=$

$$K = \frac{E}{3(1-2\mu)}$$

$$K = \frac{100}{3(1-2 \times 0.5)}$$

$$K = \frac{100}{3(1-1)}$$

4.) $F = 10$ KN, final dia $= 5.05$ m



sol^n

a.) stress $= \dfrac{\text{load}}{\text{Area}} = \dfrac{10 \text{ KN}}{\frac{\pi}{4}(d)^2}$

$= \dfrac{10}{3.14 \times (2.5)^2} = \dfrac{10}{}$

$= 0.509$ KN/m^2

b.) strain $= \dfrac{f.L - I.L}{I.L}$

$= \dfrac{5.05 - 5}{5}$

$= \dfrac{0.05}{5}$

$= 0.01$

c.) Modulus of elasticity $= \dfrac{\text{Normal stress}}{\text{Normal strain}}$

$$= \frac{0.509}{0.01} \text{ kN/mm}^2$$

$$= 50.9 \text{ kN/mm}^2$$

d.) $G = ?$

$$E = 2G(1+\mu)$$

$$50.9 = 2G(1+0.5)$$

$$\therefore G = \frac{50.9}{3}$$

$$= 16.966 \text{ kN/mm}^2$$

e.) $K = ?$

$$E = 3K(1-2\mu)$$

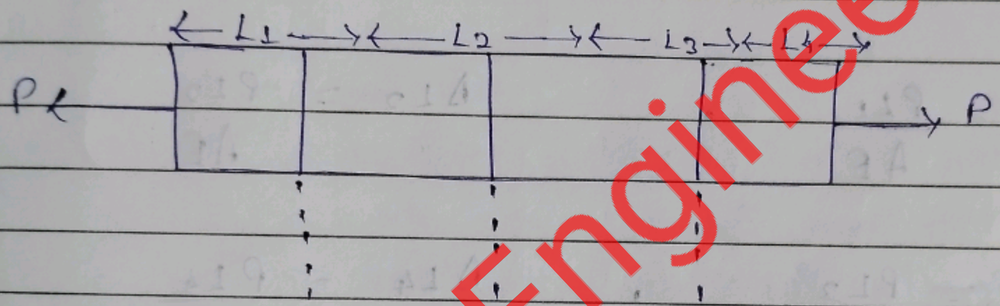
$$50.9 = 3K(1-2\times 0.5)$$

$$K = \frac{50.9}{3(1-1)}$$

$$K = \frac{50.9}{3(0)}$$

* Principal of Superposition :

When a body ~~of~~ or object is subjected to number of forces acting on it along a length of the body o object, the resultant deformation this equal to the deformation of ~~individual~~ individual object.



$$\delta L \text{ or } \Delta L = \frac{PL}{AE} = \frac{P}{AE}\left(\Sigma\left(L_1 + L_2 + \dots L_n\right)\right)$$

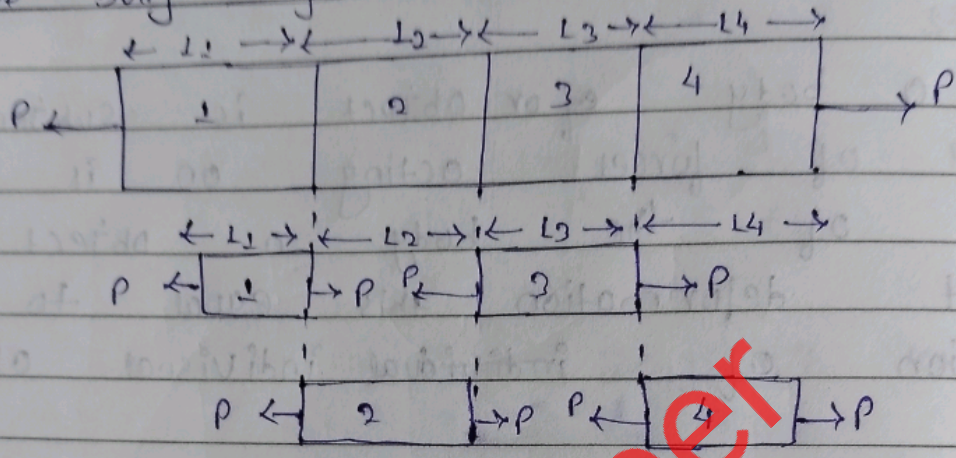
$$\sigma = \frac{P}{A} \quad , \quad \epsilon = \frac{\Delta L}{L}$$

$$E = \frac{\sigma}{\epsilon} = \frac{P/A}{\Delta L/L} = \frac{P \times L}{A \times \Delta L}$$

$$\therefore \boxed{\Delta L = \frac{PL}{AE}}$$

V.V.I

→ Free Body Diagram (FBD):



$$\Delta L_1 = \frac{PL_1}{AE}, \quad \Delta L_2 = \frac{PL_2}{AE}.$$

$$\Delta L_3 = \frac{PL_3}{AE}, \quad \Delta L_4 = \frac{PL_4}{AE}$$

Load is equal

Area & Modulus of elasticity is same

$$\delta L = \Delta L_1 + \Delta L_2 + \Delta L_3 + \Delta L_4 + \ldots \Delta L_n$$

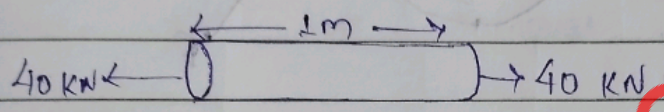
$$= \frac{PL_1}{AE} + \frac{PL_2}{AE} + \ldots \frac{PL_n}{AE}$$

V.V.I

Q. A steel rod of diameter 20 mm is subjected to a tensile force of 40 KN. The length of rod is 1m and E = 200 GPa (Gigapascal). Determine the alongation of rod.

Sol^n

Given,



$D = 20 \text{ mm}$

$P = 40 \text{ KN} = 40 \times 10^3 \text{ N}$

$L = 1 \text{ m} = 1 \times 10^3 \text{ mm}$

$E = 200 \text{ GPa} = 200 \times 10^3 \text{ N/mm}^2$

$= 2 \times 10^5 \text{ N/mm}^2$

$$A = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times 20^2 = \frac{3.14 \times \cancel{400}}{4 \times \cancel{100}}$$

$$= 314 \text{ mm}^2$$

$$\therefore \Delta L = \frac{\overset{20}{\cancel{40}} \times 10^3 \times 1 \times 10^3}{314 \times \cancel{2} \times \cancel{10^5}}$$

$$= \frac{20 \times 10}{314}$$

$$= 0.636 \text{ mm}$$

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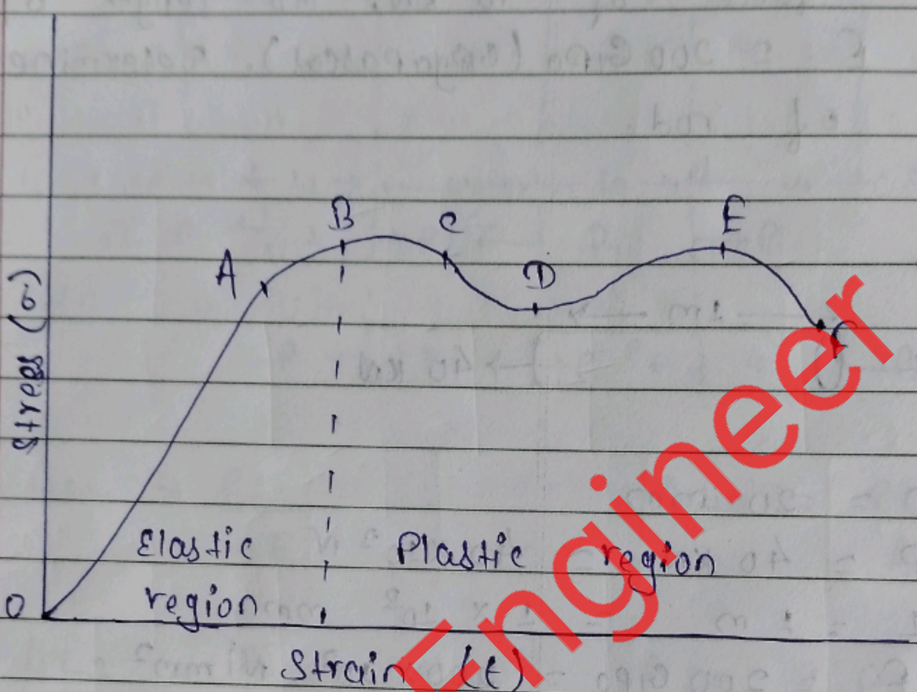
V.V.V.I

Stress vs Strain curve of mild Steel :



A → Limit of proportionality ($\sigma \propto \epsilon$, Hooke's law Valid)

B → Elastic Point

C → Upper yeild Point

D → lower yeild point

E → ultimate point

F → ~~Fractie~~ fracture Point / breaking Pt.

From O to B there is a elastic region in which object region its Position after removal of load.

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Point A :-

This is limit of proportionality up to which stress is directly proportion to strain.

Beyond Point B there is a plastic region in which object cannot region its original position of the removal of load.

Point E :-

It is an ultimate stress point at which object attain is maximum stress

Point F :-

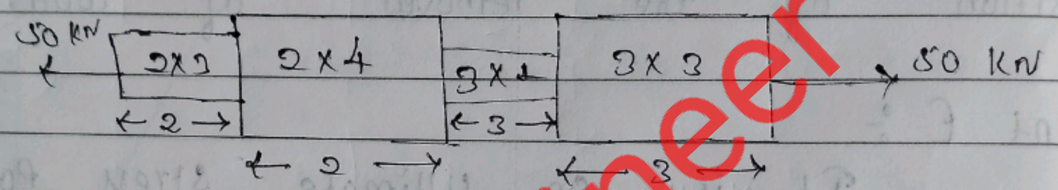
It is a failure starting beyound the point E, object get break and fracture at point C.

V. IMP

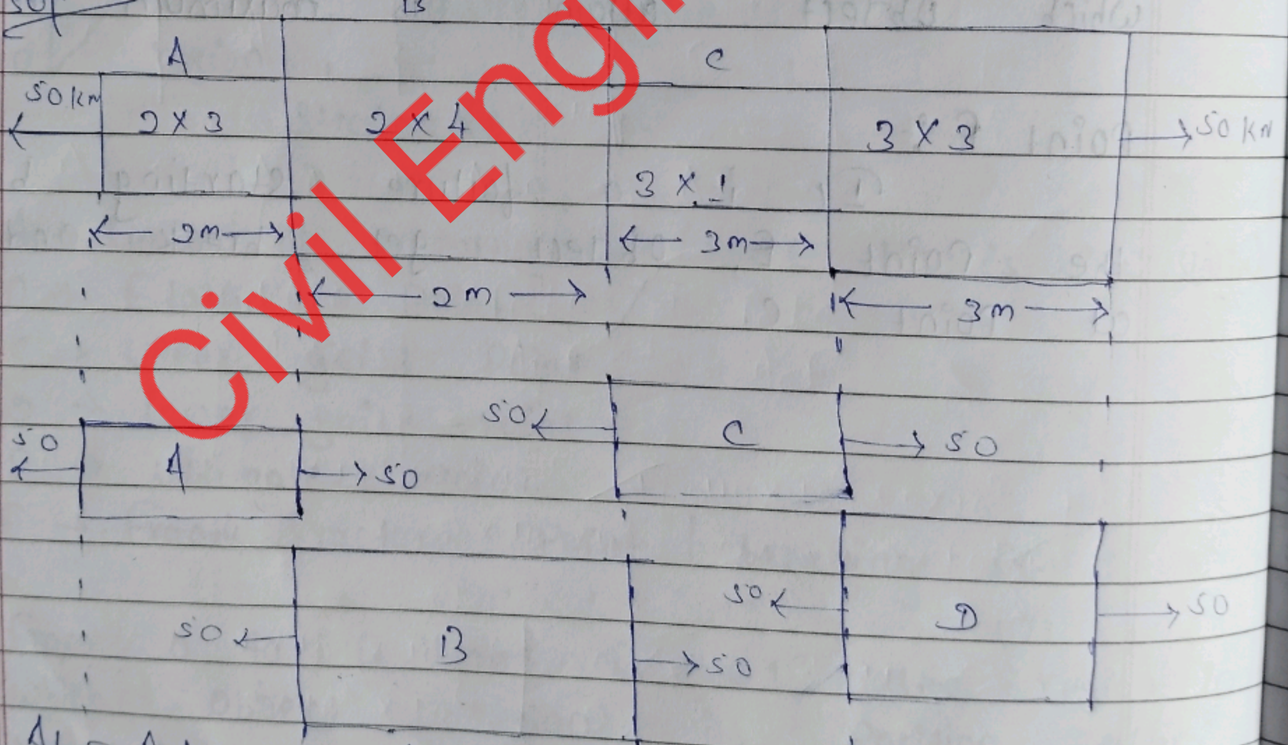
Numerical on principal of superposition

Q. Determine the stress and total elongation of the given object in which all object are of different size carrying as mention in the . Let $E = 2 \times 10^5 \ N/mm^2$

Q.)



sol^n



$$\Delta L = \Delta L_A + \Delta L_B + \Delta L_C + \Delta L_D$$

$$= \frac{P L_A}{A_A E} + \frac{P L_B}{A_B E} + \frac{P L_C}{A_C E} + \frac{P L_D}{A_D E}$$

$$\Delta L = \frac{P}{E}\left(\frac{L_A}{A_A} + \frac{L_B}{A_B} + \frac{L_C}{A_C} + \frac{L_D}{A_D}\right)$$

$$= \frac{50}{2\times10^5}\left(\frac{2}{2\times3} + \frac{2}{2\times4} + \frac{3}{2\times1} + \frac{3}{3\times3}\right)$$

$$= \frac{50\ kN}{2\times10^5\ (N/mm^2)}\left[\frac{m}{m^2}\right]$$

$$= \frac{50\times\cancel{1kN}\ 10^3 N\times mm^2}{2\times10^5\times N}\left[\frac{1}{10^3\ mm}\right]$$

$$= \frac{50}{2\times10^5}\left(\frac{2}{6} + \frac{2}{8} + \frac{3}{3} + \frac{3}{9}\right) mm$$

$$= \frac{50}{2\times10^5}\left(\frac{24+18+72+24}{72}\right) mm$$

$$= \frac{50}{2\times10^5}\times\frac{\cancel{138}\ 69}{72}\ mm$$

$$= \cancel{47.91\times10^{-5}\ mm}$$

$$= 4.791\times10^{-4}\ mm \quad \text{Ans.}$$

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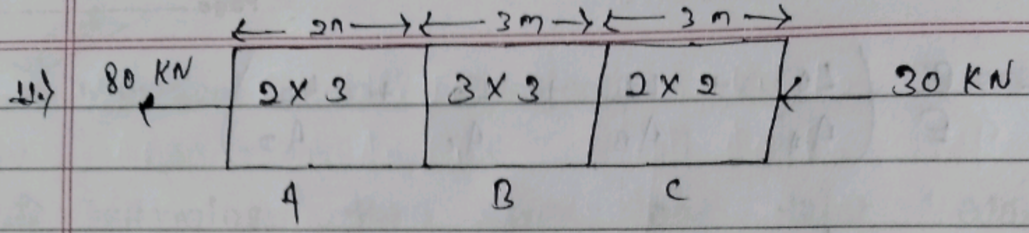
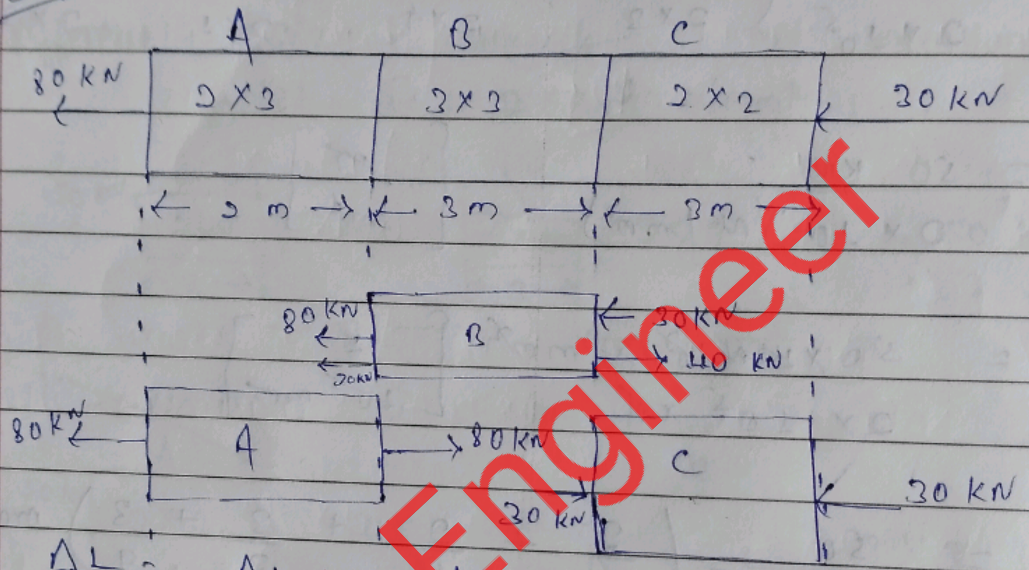
11.)



Sol^n



$$\Delta L = \Delta L_A + \Delta L_B + \Delta L_C$$

$$= \frac{P_A L_A}{A_A E} + \frac{P_B L_B}{A_B E} + \frac{P_C L_C}{A_C E}$$

$$\Delta L = \frac{80\times10^3 \times 2\times10^3}{2\times3 \times 2\times10^5\times10^6} + \frac{110\times10^3 \times 3\times10^3}{3\times3 \times 2\times10^5\times10^6} + \frac{30\times10^3\times3\times10^3}{2\times2\times2\times10^5\times10^6}$$

$$= \frac{1\times10^3\times10^3}{2\times10^5\times10^6}\left[\frac{80\times\cancel{2}}{\cancel{2}\times3} + \frac{110\times\cancel{3}}{\cancel{3}\times3} + \frac{30\times\cancel{2}}{\cancel{2}\times2}\right]$$

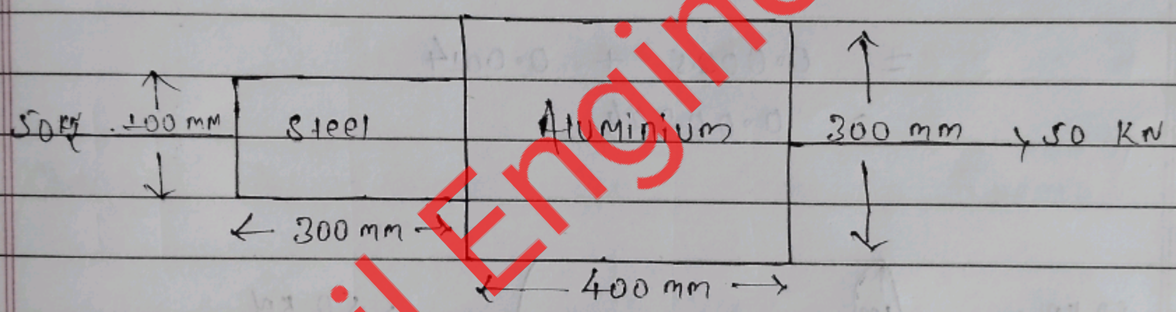
$$= \frac{10^6}{2\times10^5\times10^6}\left[\frac{80}{3} + \frac{110}{3} + \frac{30}{2}\right] \text{ mm}$$

$$= \frac{10^6}{2 \times 10^5 \times 10^6} \left(\frac{160 + 220 + 90}{6} \right) mm$$

$$= \frac{10^6 \times 470}{2 \times 10^5 \times 10^6 \times 6} \quad mm$$

$$= 3.91 \times 10^{-4} \quad mm \quad \text{Ans.}$$

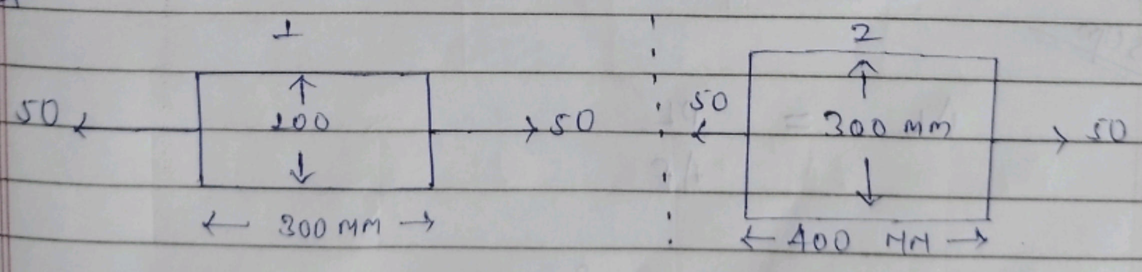
Q.



$E_s = 2 \times 10^5 \ N/mm^2$, $E_A = 1.15 \times 10^5 N/mm^2$

Calculate $\Delta L = ?$

Soln



$\Delta L = L A_1 L_1 + L_2$

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$$\frac{PL}{AE} = \frac{P_1 L_1}{A_1 E_1} + \frac{P_1 L_2}{A_2 E_2}$$

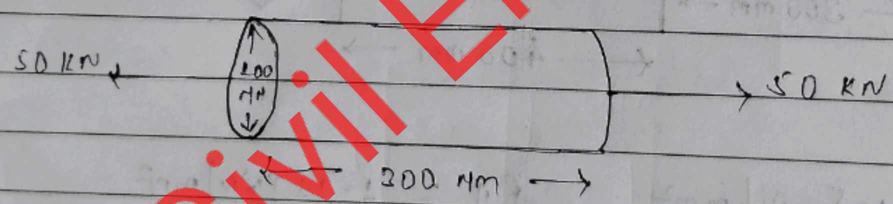
$$= \frac{\cancel{50} \times \cancel{10^3} \times \cancel{300}}{2 \times \cancel{10^5} \times 100 \times \cancel{300}} + \frac{\cancel{50} \times \cancel{10^3} \times \cancel{400}}{\cancel{2} \times 1.15 \times \cancel{10^5} \times \cancel{400} \times 300}$$

$$= \frac{1}{400} + \frac{1}{690}$$

$$= 0.0025 + 0.0014$$

$$= 0.00394$$

Q.



$E = 2 \times 10^5 \ N/mm^2$

Soln

$$\Delta L = \frac{PL}{AE}$$

$$= \frac{50 \times 10^3 \times 300}{\frac{\pi}{4} \times (100)^2 \times 2 \times 10^5}$$

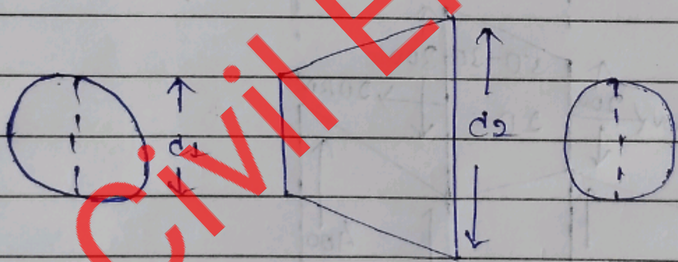
$$= \frac{50 \times \cancel{10^3} \times \cancel{300}}{7850 \times \cancel{10^5} \times 2}$$

$$= \frac{50 \times 3}{7850 \times 2} \text{ mm}$$

$$= \frac{3}{314} \text{ mm}$$

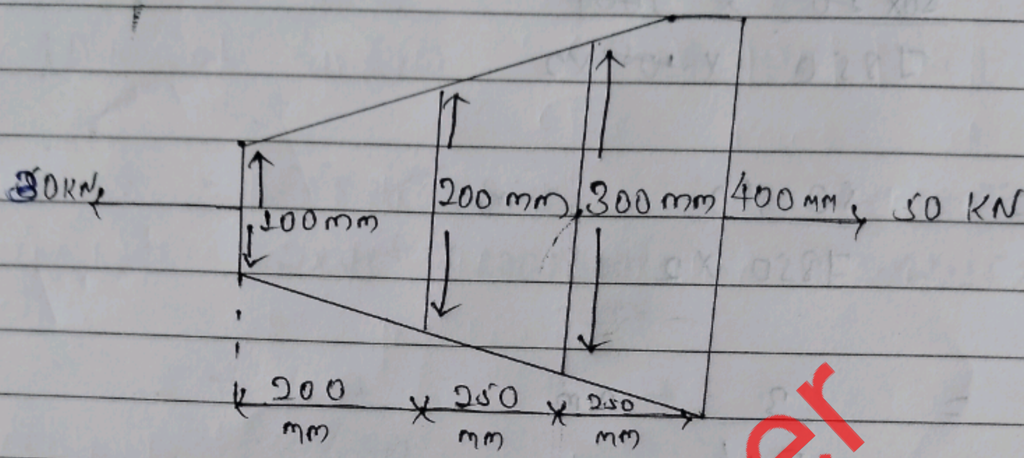
$$= 9.554 \; \cancel{kN/mm^2}$$

*. Change in diameter for circular Tapered section



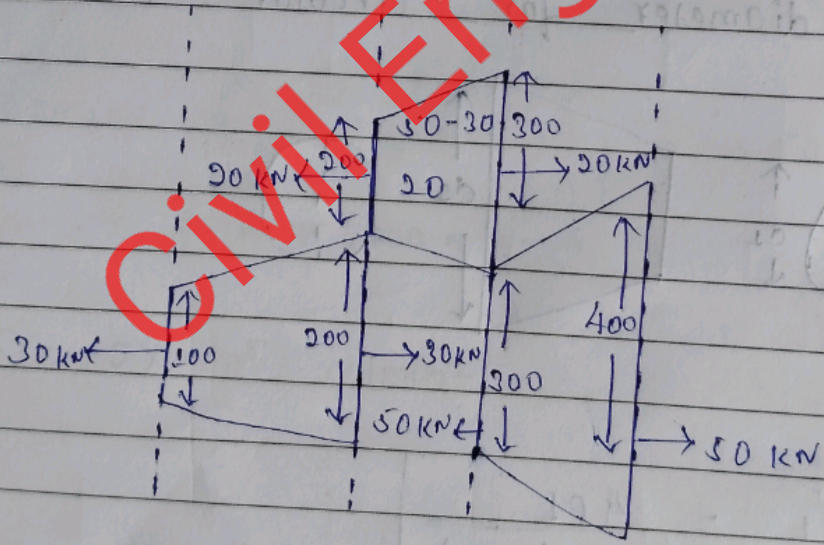
$$\boxed{\Delta L = \frac{4PL}{\pi E (d_1 d_2)}}$$

Q.



$E = 2 \times 10^5 \ N/mm^2$

Soln



$\Delta L = L_A + L_B + L_C$

$$\frac{4PL}{\pi E(d_1 d_2)} = \frac{4 P_A L_A}{\pi E(d_1 d_2)} + \frac{4 P_B L_B}{\pi E(d_1 d_2)} + \frac{4 P_C L_C}{\pi E(d_1 d_2)}$$

$$\Delta L = \left(\frac{4\times 30\times 10^3\times 200}{\pi\times 2\times 10^5\times 100\times 200}\right) + \left(\frac{4\times 20\times 10^3\times 250}{\pi\times 2\times 10^5\times 200\times 300}\right) + \left(\frac{4\times 50\times 10^3\times 250}{\pi\times 2\times 10^5\times 300\times 400}\right)$$

$$= \frac{4\times 10^3}{\pi\times 2\times 10^5}\left(\frac{30\times 200}{100\times 200} + \frac{20\times 250}{200\times 300} + \frac{50\times 250}{300\times 400}\right)$$

$$= \frac{4\times 10^3}{\pi\times 2\times 10^5}\left(\frac{3}{10} + \frac{50}{200} + \frac{125}{1200}\right)$$

$$= \frac{4\times 10^3}{3.14\times 2\times 10^5}\left(\frac{157}{240}\right)$$

$$= \frac{4\times 10^3\times 157}{3.14\times 2\times 10^5\times 240}$$

$$= 4.167\times 10^{-3}$$

CH- 06

CENTRE OF GRAVITY

*. centre of gravity :- (C.G) : [X̄] [Ȳ]

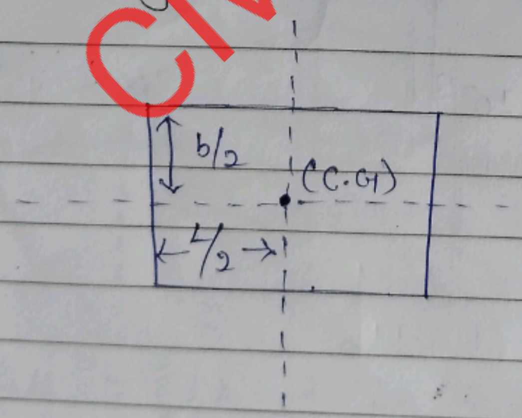
The Point at which the whole weight of the body or object ~~act~~ act irrespect of its Position is known as centre of gravity.

*. centroid :-

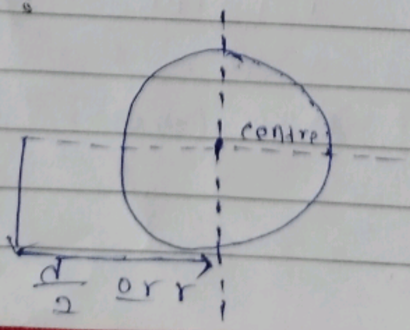
In the Plane figure (square, circle, etc) have only area but no mass. The centre of area of that object is called centroid.

↳ Centre of gravity :-

a.) Rectangle square :-

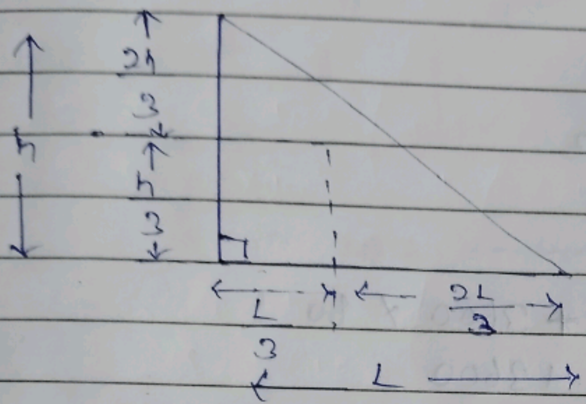


b.) circle :-



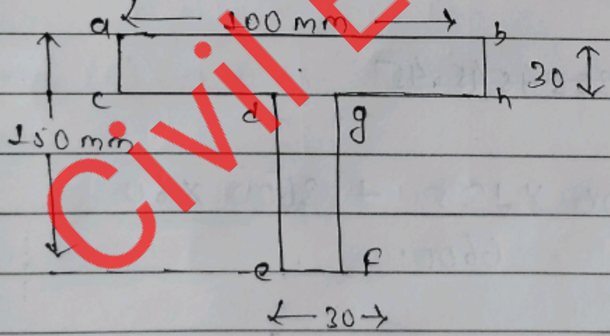
$C.G = \frac{Diameter}{2}$

(c) Right angled ~~traing~~ triangle :



$\frac{L}{3}$ & $\frac{h}{3}$ from perpendicular side.

Q. Determine the centre of gravity of the given T-section about yy axis. symmetrical



soln

$$[A\bar{Y} = a_1\bar{y}_1 + a_2\bar{y}_2 + \ldots\ a_n\bar{y}_n]$$

For area (A) $= a_1 = 100 \times 30$ $= 3000\ mm^2$, $y_1 = 120 + \frac{30}{2}$ $= 135$

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for area (2) → $a_2 = 120 \times 30$, $\bar{y}_2 = \frac{120}{2} = 60$

$= 3600\ mm^2$

$$\Rightarrow \bar{y} = \frac{a_1\bar{y}_1 + a_2\bar{y}_2}{A}$$

$$\Rightarrow \bar{y} = \frac{3000 \times 135 + 3600 \times 60}{3000 + 3600}$$

$$\Rightarrow \bar{y} = \frac{3000 \times 135 \times \cancel{3600} \times \cancel{60}^{10}}{\cancel{6600}_{11}}$$

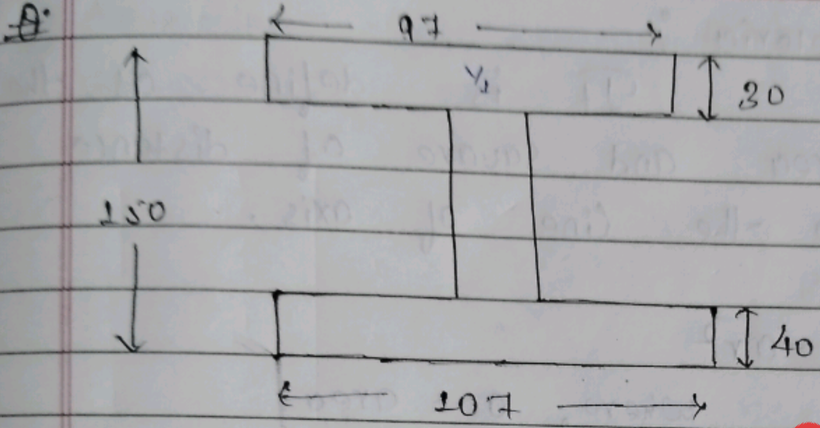
$$= \cancel{1325 4545.45}$$

$$\Rightarrow \bar{y} = \frac{3000 \times 135 + 3600 \times 60}{6600}$$

$$= \frac{1035}{11}$$

$$= 94.0909$$

Q.



Sol^n

$$\left[A\bar{Y} = a_1\bar{Y} + a_2\bar{Y}_2 + a_3\bar{Y}_3\right]$$

For area (A) $= 97 \times 30$, $\bar{y}_1 = 120 + \frac{30}{2}$

$= 2700\ mm^2$ $= 135$

For area (A) $\Rightarrow$ B =

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#. Moment of Interia :

It is define as the product of area and squore of distance perpendicular to the line of axis.

$$M.O.I \ (I) = ar^2$$

where, a = area

r = $\perp r$ distance from axis

→ Theorem of determination of (M.O.I) :

- Theorem of perpendicular axis
- Therem of parallel axis

(i) Theorem of perpendicular axis :

It state that if I_{xx} + I_{yy} be the moment of interia of a plane section about to perpendicular axis meeting at O (center point), the moment of interia about ZZ axis perpendicular to plane is as

$$I_{zz} = I_{xx} + I_{yy}$$

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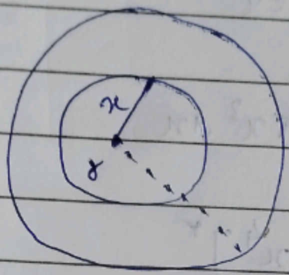
*. Determine the moment of interia of a circular section of radius (r)



Radius of circle = r

so, Area of circle = πr^2

Take a strip at dist. x from centre.

so,

Area of distance 'x' = πr^2

Now,

differentiate of Area of x

$$\Rightarrow \frac{dA}{dx} = [\pi x^2] \Rightarrow so, \ \frac{dA}{dx} = 2\pi x$$

$$[dA = 2\pi x \, dx] \longrightarrow (i)$$

on integrate above eqn - (i)

$$\Rightarrow A = \pi x^2$$

Now we know that,

$$I = a r^2$$

$$I = (2\pi x \, dx) \times x^2$$

$$= 2\pi x^3 dx$$

$$dI = 2\pi x^3 \, dx$$

On, integrate above eqn

$$\Rightarrow \int dI = \int_0^r 2\pi x^3 \, dx$$

$$\Rightarrow I = 2\pi \left[\frac{x^4}{4} \right]_0^r$$

$$\Rightarrow I = 2\pi \left[\frac{r^4}{4} - \frac{0^4}{4} \right]$$

$$\Rightarrow I = \left(\frac{2\pi r^4}{4} \right) = \frac{\pi r^4}{2}$$

$$\therefore I_{xx} = \frac{\pi r^4}{2} = I_{yy}$$

Or,

$$r = \frac{D}{2}$$

Also,

$$I_{xx} = I_{yy} = \frac{\pi}{2} \left(\frac{D}{2} \right)^4$$

$$\boxed{\cancel{I_{xx} = I_{yy}} \; I_{zz} = \frac{\pi D^4}{32}}$$

Now, by ⊥r axis theorem,

$$I_{zz} = I_{xx} + I_{yy}$$

$$I_{xx} = I_{yy}$$

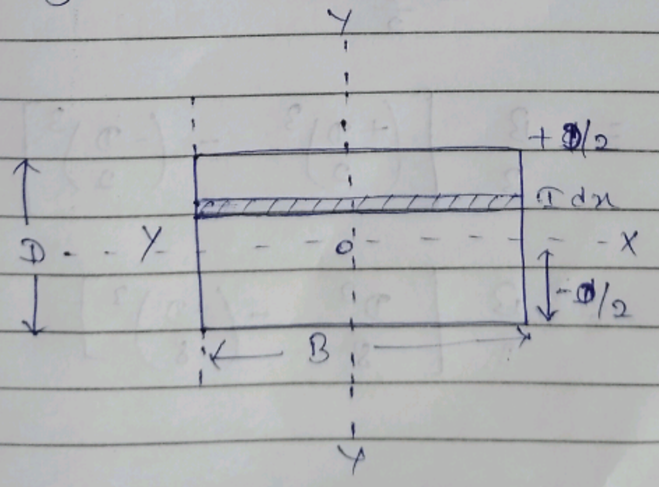
$$I_{zz} = I_{xx} + I_{xx} = 2 I_{xx}$$

$$\therefore \quad I_{xx} = \frac{I_{zz}}{2} = I_{yy}$$

$$\therefore \quad I_{xx} = I_{yy} = \frac{1}{2}\left(\frac{\pi D^4}{32}\right)$$

$$\boxed{I_{xx} = I_{yy} = \frac{\pi D^4}{64}}$$

b) Rectangle of side $B \times D$:



Now,

Area of rectangle = $B \times D$

Take a thin strip of rect. of width 'dx'

$$dA = B \times dx$$

As we know that,

$$I = A.r^2 = dA \times x^2 = B \times dx \times x^2$$

$$\therefore \quad I = Bx^2\,dx$$

On integrate $I = Bx^2\,dx$

$$\int I = \int_{-D/2}^{+D/2} Bx^2\,dx$$

$$I_{xx} = B\left[\frac{x^3}{3}\right]_{-\frac{D}{2}}^{+D/2}$$

$$I_{xx} = \frac{B}{3}\left[\left(+\frac{D}{2}\right)^3 - \left(-\frac{D}{2}\right)^3\right]$$

$$I_{xx} = \frac{B}{3}\left[\frac{D^3}{8} - \left(-\frac{D}{8}\right)^3\right]$$

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$$I_{xx} = \frac{B}{3}\left[\frac{D\,D^3}{8}\right] = \frac{BD^3}{12}$$

$$\therefore \boxed{I_{xx} = \frac{BD^3}{12}}$$